New compactness estimates for aggregation-diffusion equations

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I will present two results dealing with the passage to the limit in aggregation-diffusion equations where obtaining standard compactness estimates is difficult. The first result, obtained in collaboration with C. Elbar and B. Perthame, concerns the kinetic derivation of the degenerate Cahn-Hilliard equation from a certain nonlocal partial differential equation. The challenge here is that all necessary a priori estimates can only be obtained for the nonlocal quantities, providing almost no information about the limiting solution itself. We introduce a novel condition on the kernel that allows us to exploit the available nonlocal a priori estimates. The second result, obtained in collaboration with J. A. Carrillo and Y. Salmaniw, concerns the existence (and uniqueness) of solutions to aggregation-diffusion equations where the kernel is only bounded and integrable, for instance, a characteristic function of a ball or a cube. Here, we take advantage of the gradient flow structure in a novel way, utilizing the dissipation of free energy and equiintegrability to control the gradient of the solution. This second work is particularly important in ecology, where the case of a characteristic function of a cube is widely used as a toy model to study the dynamics of populations.