

Testing topological conjugacy of time series

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We consider a problem of testing topological conjugacy of two trajectories coming from dynamical systems (X, f) and (Y, g) . More precisely, given $x_1, \dots, x_n \subset X$ and $y_1, \dots, y_n \subset Y$ such that $x_{i+1} = f(x_i)$ and $y_{i+1} = g(y_i)$ (for some unknown maps f and g), as well as $h : X \rightarrow Y$, we deliver a number of tests to check if the corresponding trajectories of f and g are topologically conjugated via h . The values of the tests are close to zero for systems conjugate by h and large for systems that are not. For our main developed method, ConjTest, the convergence of the test values, in case when sample size goes to infinity, is established.

We provide numerical examples indicating scalability and robustness of the presented methods. In addition, we show how the presented method gives rise to a test of sufficient embedding dimension, mentioned in Takens' embedding theorem. Finally, we include a proof-of-concept study using the presented methods to search for an approximation of the homeomorphism conjugating given systems.

The talk is based on a joint work with Paweł Dłotko (IM PAN, Dioscuri Centre in TDA) and Michał Lipiński (Institute of Science and Technology Austria).

References

- [1] P. Dłotko, M. Lipiński, J. Signerska-Rynkowska, *Testing topological conjugacy of time series*, arXiv:2301.06753 (2023)