Using an invariant knot of a flow to detect additional invariant structure

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Consider a continuous flow in $\mathbb{R}^3$ and suppose $N \subseteq \mathbb{R}^3$ is a compact 3–manifold such that the trajectories of the flow either cross ∂N transversally or bounce off it from the outside. Suppose we know that there exists an invariant knot or link K in the interior of N and want to look for additional invariant structure inside N. The following theorem holds: if K is contractible (in N) and nontrivial (in the sense of knot theory), then every neighbourhood U of K contains a point $p \in U \setminus K$ such that the trajectory through p is entirely contained in N. In other words, the presence of a contractible invariant knot in N forces the existence of additional invariant structure in N which, moreover, passes arbitrarily close to K.

The proof of this result makes use of a "coloured handle" theory which may be of independent interest to study flows in 3-manifolds. The goal of the talk is to introduce these tools and give an idea of how to prove the theorem using them.