

Stability of the weak* fixed point property in ℓ_1

Lukasz Piasecki

Instytut Matematyki, Uniwersytet Marii Curie-Skłodowskiej

Pl. Marii Curie-Skłodowskiej 1, 20-031 Lublin, Poland

lukasz.piasecki@mail.umcs.pl

A nonempty, bounded, closed and convex subset C of a Banach space X has the *fixed point property* (briefly, FPP) if each nonexpansive mapping $T : C \rightarrow C$ (i.e., $\|T(x) - T(y)\| \leq \|x - y\|$ for all $x, y \in C$) has a fixed point. A dual space X^* is said to have the $\sigma(X^*, X)$ -*fixed point property* (briefly, $\sigma(X^*, X)$ -FPP) if every nonempty, convex, $\sigma(X^*, X)$ -compact set $C \subset X^*$ has the FPP. We say that X^* enjoys the *stable* $\sigma(X^*, X)$ -FPP if there exists $\gamma > 1$ such that Y^* has the $\sigma(Y^*, Y)$ -FPP whenever the Banach–Mazur distance $d(X, Y) < \gamma$.

We give several characterizations of all preduals X of ℓ_1 such that X^* has the stable $\sigma(\ell_1, X)$ -FPP.

References

- [1] E. Casini, E. Miglierina, L. Piasecki, *Separable Lindenstrauss spaces whose duals lack the weak* fixed point property for nonexpansive mappings*, *Studia Math.* **238** (2017), 1–16.
- [2] E. Casini, E. Miglierina, L. Piasecki, R. Popescu, *Weak* fixed point property in ℓ_1 and polyhedrality in Lindenstrauss spaces*, *Studia Math.* **241** (2018), 159–172.
- [3] E. Casini, E. Miglierina, L. Piasecki, R. Popescu, *Stability constants of the weak* fixed point property in the space ℓ_1* , *J. Math. Anal. Appl.* **452** (2017), 673–684.
- [4] L. Piasecki, *On Banach space properties that are not invariant under the Banach–Mazur distance 1*, *J. Math. Anal. Appl.* **467** (2018), 1129–1147.
- [5] L. Piasecki, *On ℓ_1 -preduals distant by 1*, *Ann. Univ. Mariae Curie-Skłodowska Sect. A* **72** (2018), 41–56.
- [6] E. Casini, E. Miglierina, L. Piasecki, *Subspaces of separable L_1 -preduals: W_α everywhere*, arXiv:2401.04819