Positive solutions to nonhomogeneous quasilinear problems with singular and supercritical nonlinearities

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In recent years, there has been a growing interest in nonlinear singular elliptic PDEs. We study the existence of nonnegative solutions for the following quasilinear and singular elliptic problems with supercritical nonlinearity

$$\begin{cases} -\Delta_p u - \Delta_q u = \lambda \dfrac{h(x)}{u^\gamma} + u^\theta, \ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where Ω is an open, bounded subset of $\mathbb{R}^N (N \geq 3)$ with C^2 boundary, h is a positive real-valued function, $1 < p < q < \infty$ and λ, θ, γ are positive parameters. Our motivation for this problem is taken from [1], where the authors considered the following problem

$$\begin{cases} -\operatorname{div}(M(x)\nabla u) = \lambda u^{-\gamma} + u^\theta, \ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

Our objective is to investigate problem (1), focusing on the impact of singular and supercritical nonlinearities on the right-hand side, alongside the nonhomogeneous operator. In particular, for supercritical cases, i.e., $\theta \geq q^* - 1$, we prove the existence of solutions in a weak sense. To demonstrate the existence of a weak solution, we utilize the method of sub and supersolution.

References

[1] Boccardo, L.: *A Dirichlet problem with singular and supercritical nonlinearities*, Nonlinear Analysis: Theory, Methods And Applications **75** (2012), 4436-4440. doi: 10.1016/j.na.2011.09.026.