Weak* fixed point property in ℓ_1

Enrico Miglierina
DIMSEFA, Università Cattolica, Milano, Italy
enrico.miglierina@unicatt.it

Emanuele Casini
Dipartimento di Scienza e Alta Tecnologia, Università dell'Insubria, Como, Italy
emanuele.casini@uninsubria.it

Łukasz Piasecki
Instytut Matematyki, Uniwersytet Marii Curie-Skłodowskiej, Lublin, Poland
lukasz.piasecki@mail.umcs.pl

A typical problem in the study of fixed point property for nonexpansive mappings is to characterize the spaces enjoying this property within a specific class of Banach spaces (see, e.g., [6, 5]).

We recall that the space X^* is said to have the weak* fixed point property (briefly, w^*-FPP) if for every nonempty, convex, w^*-compact subset C of X^*, every nonexpansive mapping (i.e., a mapping $T : C \to C$ such that $\|T(x) - T(y)\| \leq \|x - y\|$ for all $x, y \in C$) has a fixed point.

In this talk we study the w^*-fixed point property for the space ℓ_1 endowed with the weak* topologies generated by different preduals X. First, we provide some sufficient conditions for w^*-FPP in ℓ_1 based on the presence of particular subspaces in the predual X of ℓ_1. Then, we completely characterize w^*-FPP in ℓ_1 in terms of the existence of specific quotients of the predual space X. A key tool of our results is a detailed study of the hyperplanes of the space c of convergent sequences ([1]). Moreover, also a particular class of ℓ_1-preduals, the spaces of affine functions on Choquet simplex, plays an important role. Finally, we show that, in our characterizations, the existence of suitable quotients in the preduals X of ℓ_1 cannot be replaced by that of subspaces in X.

This talk is based on a series of papers written jointly with Emanuele Casini and Łukasz Piasecki ([2, 3, 4]).

The speaker is partially supported by INdAM - GNAMPA Project", codice CUP E53C23001670001.

References

[1] E. Casini, E. Miglierina and Ł. Piasecki, *Hyperplanes in the space of convergent sequences and preduals of ℓ_1*, Canad. Math. Bull. **58** (2015), 459–470.

[2] E. Casini, E. Miglierina and Ł. Piasecki, *Separable Lindenstrauss spaces whose duals lack the weak* fixed point property for nonexpansive mappings*, Studia Math. **238** (2017), 1–16.

[3] E. Casini, E. Miglierina and Ł. Piasecki, *Weak* fixed point property and the space of affine functions*, Proc. Amer. Math. Soc. **149** (2021), 1613–1620.

[4] E. Casini, E. Miglierina and Ł. Piasecki, *Explicit models of ℓ_1-preduals and the weak* fixed point property in ℓ_1*, Topol. Methods Nonlinear Anal. **63** (2024), 39–51.

[5] P. N. Dowling and C. J. Lennard, *Every nonreflexive subspace of $L_1[0,1]$ fails the fixed point property*, Proc. Amer. Math. Soc. **125** (1997), 443–446.

[6] B. Maurey, *Points fixes des contractions de certain faiblement compacts de L^1*, in: Seminaire d'Analyse Fonctionelle (Paris), Exposé **VIII**, École Polytechnique, Centre de Mathematiques, Paris, 1980–1981.