Realization of prescribed set of minimal periods for the Morse-Smale diffeomorphisms

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A given self-map $f\colon M \to M$ of a compact manifold determines the sequence $\{l_n = L(f^n)\}$, of the Lefschetz numbers of its iterations. We consider its dual sequence $\{a_n(f)\}_{n=1}^{\infty}$ given by the Möbius inversion formula $a_n(f) = \frac{1}{n}\sum_{d|n}\mu(d)L(f^{\frac{n}{d}})$. The set $\mathcal{A}(f) = \{n : a_n(f) \neq 0\}$ is called the set of algebraic periods. We show that for every finite subset of $\mathcal{A} \subset \mathbb{N}$ of natural numbers there exist an orientable surface S_{g} of genus g and Morse-Smale diffeomorphism f of this surface such that $\mathcal{A}(f) = \mathcal{A}$. For any map from this class, as well as for every transversal map homotopic to it, this implies the existence of points with minimal period equal to $n \in \mathcal{A}$, n-odd. We also show that in the case when f reverses the orientation, there are restrictions on $\mathcal{A}$. Our result provides a solution to questions existing in the literature. During this talk we outline the topological part of this project based on the Nielsen-Thurston classification theorem.

The algebraic part will be presented also at the conference.