Bistability in a one-dimensional model of a two-predators-one-prey population dynamics system

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We study the classical two-predators-one-prey model, introduced in [1]

$$\dot{y}_i = m_i \frac{s-\lambda_i}{s+a_i} y_i, \qquad i = 1, 2, \qquad \text{(predators)}$$
$$\dot{s} = \left(1 - s - \frac{y_1}{s+a_1} - \frac{y_2}{s+a_2}\right) s \qquad \text{(prey)}$$

with all constants a_i, λ_i and m_i being positive. It is shown in [2] that for a broad range of parameter values the considered system exhibits a strong contraction in the $(y_1 + y_2)$-direction in which case its dynamics can be approximated by the one-dimensional map given by a one-dimensional bimodal map:

$$x_{n+1} = f(x_n) = b + x_n - \frac{k}{1 + e^{x_n}}$$

The map f has a negative Schwartzian derivative and, therefore, the considered system could have at most two attracting orbits (periodic or chaotic). We find where short-period regimes can be observed (in particular, we prove that there might be only one period-2 solution).

We describe analytically the structure of bifurcations of the map. Taking this mechanism into account, one can easily detect parameter regions where cycles with arbitrary high periods or chaotic attractors with arbitrary high numbers of bands coexist pairwise.

The domains where two attractors coexist are found out numerically.

References

[1] A. S. Petrov, G. J. Söderbacka, *Review on the behaviour of a many predator - one prey system*, Dinamicheskie sistemy. **9(37)** (2019), 273 - 288.

[2] T. Eirola, A. V. Osipov, G. J. Söderbacka, *The concept of spectral dichotomy for linear difference equations II*, Research reports A, Helsinki University of Technology, **386** (1996).