Bôcher type theorem for gradient perturbed Lévy operators: supercritical and subcritical cases

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A classical Bôcher's theorem asserts that any positive harmonic function (with respect to the Laplacian) in the punctured unit ball can be expressed as a linear combination of the Green function for the unit ball and a positive function that is harmonic in the whole unit ball. This theorem expresses one of the fundamental results in the theory of isolated singularities and it can be viewed as a statement on the asymptotic behavior of positive harmonic functions near their isolated singularities. We present a generalization of this results to Lévy operators perturbed by spatially inhomogeneous gradient operator. We propose a probabilistic potential theory approach which allows us to achieve the main result without any restrictions on the Lévy operator. In particular our result covers gradient perturbed fractional Laplacians with any index of stability bounded between zero and two - the method is therefore applicable to subcritical and supercritical cases.

References

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