Polychromatic Surface Plasmons in Kerr-Nonlinear Media

Maximilian Hanisch
Institute of Mathematics, Martin-Luther-Universität, Halle-Wittenberg, Germany
maximilian.hanisch@mathematik.uni-halle.de

We study Maxwell's equation in a one-dimensional geometry on the interface between two media with a nonlinear and non-local (in time) dependence of the electric displacement field $\mathcal{D}$ on the electric field $\mathcal{E}$ of the form

$$\begin{aligned}\mathcal{D}(x,y,t) = \epsilon_0 \mathcal{E}(x,y,t) + \epsilon_0 \int_{\mathbb{R}} \chi^{(1)}\left(x, t-s\right) \mathcal{E}(x,y,s) \mathrm{d}s \\ + \epsilon_0 \int_{\mathbb{R}^3} \chi^{(3)}\left(x, t-s_1, t-s_2, t-s_3\right) \\ \left(\left(\mathcal{E}(x,y,s_1) \cdot \mathcal{E}(x,y,s_2)\right) \mathcal{E}(x,y,s_3)\right) \mathrm{d}(s_1, s_2, s_3).\end{aligned}$$

The monochromatic ansatz

$$\mathcal{E}(x,t) = e^{-\mathrm{i}\omega t} E(x) + e^{\mathrm{i}\overline{\omega} t} \overline{E(x)}$$

lets us study an associated spectral problem, which is well understood [1]. In the nonlinear setting however, this ansatz produces also time dependences with higher frequencies. These cannot be compensated by the linear terms of the equation and are thus commonly neglected in the physics literature.

To treat the full nonlinear problem, we introduce a polychromatic ansatz for a solution as an infinite series over integer multiples of the frequency ω and show the existence of such solutions in a physically meaningful setting.

References

[1] Spectrum of the Maxwell equations for a flat interface between homogeneous dispersive media, Brown, M., Dohnal, T., Plum, M. and Wood, I., arXiv preprint arXiv:2206.02037, 2022.