# Global compactness result and multiplicity of solutions for a class of critical exponent problems in the hyperbolic space

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Consider the problems of the type

$$-\Delta_{\mathbb{B}^N} u - \lambda u = a(x)|u|^{2^*-2}u + f(x) \quad \text{in } \mathbb{B}^N, \quad u \in H^1(\mathbb{B}^N),$$

where $\mathbb{B}^N$ denotes the ball model of the hyperbolic space of dimension $N \geq 4$, $2^* = \frac{2N}{N-2}$, $\frac{N(N-2)}{4} < \lambda < \frac{(N-1)^2}{4}$ and $f \in H^{-1}(\mathbb{B}^N)$ $(f \not\equiv 0)$ is a non-negative functional in the dual space of $H^1(\mathbb{B}^N)$. The potential $a \in L^\infty(\mathbb{B}^N)$ is assumed to be strictly positive, such that $\lim_{d(x,0)\to\infty} a(x) = 1$, where $d(x,0)$ denotes the geodesic distance. For $f = 0$ and $a \equiv 1$, profile decomposition was studied by Bhakta and Sandeep in [1]. However, due to the presence of the potential $a(.)$, an extension of profile decomposition to the present set-up is highly nontrivial. It requires several delicate estimates and geometric arguments concerning the isometry group (Möbius group) of the hyperbolic space. The result we achieved generalizes the profile decomposition. Further, using the decomposition result, we derived various energy estimates involving the interacting hyperbolic bubbles and hyperbolic bubbles with localized Aubin-Talenti bubbles. Finally, combining these estimates with topological and variational arguments, we established a multiplicity of positive solutions in the cases: $a \geq 1$ and $a < 1$ separately. The equation studied can be thought of as a variant of a scalar-field equation with

a critical exponent in the hyperbolic space, although such a critical exponent problem in the Euclidean space $\mathbb{R}^N$ has only a trivial solution when $f \equiv 0$, $a(x) \equiv 1$ and $\lambda < 0$.

## References


[1] M. Bhakta, K. Sandeep: Poincaré-Sobolev equations in the hyperbolic space, *Calc. Var. Partial Differential Equations*, 44, 247–269(2012).