Approach to compactness in normed spaces through seminorms

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We present a new attitude to the (pre-)compactness in normed spaces. We introduce the notion of an equinormed set using a suitable family of semi-norms on the given normed space, satisfying some natural conditions. Having some freedom of choice of the set of semi-normes that is used to define equinormed sets, we may prove known compactness criteria (e.g. Arzelà-Ascoli theorem in spaces $C(X, \mathbb{R})$ or in sequence spaces l^p for $p \in [1, +\infty)$) as well as such that were unknown until recently (e.g. in the spaces $BV([0,1])$ of functions of bounded variation on compact interval or $Lip(X, \mathbb{R})$ of Lipshitz continuous functions).