On solvability of nonlinear equations driven by coercive operators

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Let E be a separable reflexive Banach space. Given an operator $A : E \to E^*$, a fixed element $b \in E^*$ we will consider the solvability of

$$A(u) = b \tag{1}$$

under the following types of assumptions:

a) A is weakly continuous, potential with a coercive potential;

b) A is continuous and coercive or else continuous, potential with a coercive potential;

c) A is continuous, potential and the following inequality is satisfied: there is a number $R > 0$ such that

$$\langle A(u) - b, u \rangle \geq 0 \text{ for } \|u\| = R.$$

In order to examine the solvability of (1) we will exploit the Weierstrass-Tonelli Theorem together with the Galerkin type approximation in case a) and techniques related to the existence of generalized solutions in case b). For the case c) we will utilize Galerkin approximations with nonlinear programming techniques pertaining to the Karush-Kuhn-Tucker Theorem.

Applications will be given to:
a) the fourth order elastic beam equation with rigidly fasten ends;
b) and c) the Dirichlet problem governed by the competing (p, q) −Laplacian.

References

[1] M. Galewski, D. Motreanu, *Competing operators and problems with unbounded weight.* in preparation.

[2] M. Galewski, D. Motreanu, *On variational competing (p, q)−Laplacian Dirichlet problem with gradient depending weight.* Appl. Math. Lett. 148C, Article ID 108881, 7 p. (2024).