Fixed point index for multivalued maps on finite spaces

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Quite recently, the Lefschetz fixed point theorem for acyclic-valued maps $F : X \multimap X$ which are susc or slsc has been proved for X being finite T_0 topological space, see [1]. The authors used the correspondence between finite spaces and posets due to Aleksandroff. In particular, the order complex $K(X)$ consisting of all chains in the poset is a simplicial complex weekly homotopically equivalent to X. Every continuous map $f : X \to X$ induces a simplicial map $K(f) : K(X) \to K(X)$. Similarly, a multivalued acyclic map $F : X \multimap X$ gives rise to a chain map $\varphi : C_*(K(X)) \to C_*(K(X))$. and the Lefschetz numbers of the maps are the same.

Thus we present the notion of a 'local' Lefschetz number on a chain level as in H. Hopf paper [3]. This was the main idea of the notion of index systems in [4], where all the standard properties of the fixed point index were proved. Therefore we obtain a 'local' Lefschetz fixed point theory.

Let us note that another local invariant - Conley index, has been defined recently in [2].

References

[1] Barmak, J.M., Mrozek M., Wanner, T., *A Lefsschetz fixed point theorem for multivalued maps of finite spaces*, Math.Z. (2020), 294, 1477-1497.

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[3] Hopf, H., *Uber die algebraische Anzahl for Fixpunkten*, Math. Z. (1929) 29, 429-462.

[4] Siegberg H.W., Skordev, G., *Fixed point index and chain approximations*, Pac. J. Math. (1982) 102, 455-486.