Analyzing Multifiltrations Using Multiparameter Discrete Morse Theory

Guillaume Brouillette
Faculté des sciences, Université de Sherbrooke, Sherbrooke, Canada
`Guillaume.Brouillette2@USherbrooke.ca`

Multifiltrations, or multiparameter filtrations, arise in various contexts, notably in topological data analysis. Essentially, a multifiltration of a space X captures its evolution with respect to possibly many parameters. As they vary, we track the evolution of X through the multipersistent homology induced by the filtration, which describes the homological changes in X. In many cases, a multifiltration can be represented as a nested sequence of sublevel sets of a vector function $f : X \to \mathbb{R}^k$, called a multifiltering function.

In this presentation, we show how to get a better grasp of such functions using multiparameter discrete Morse (`mdm`) theory, which is an extension of Morse-Forman theory to vector-valued functions. Notably, we see that a multifiltering function can be associated to a `mdm` function. Also, we see link the critical simplices of this `mdm` function to the concept of Pareto set of a (smooth) vector map. Finally, we present some practical applications in data analysis.