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Juliusz Schauder Medal Awarding Ceremony

Laureate: Thomas Bartsch

Juliusz Schauder Prize Ceremony

Winners: Bartosz Bieganowski, Wojciech Górny and Jakub Skrzeczkowski

Jubilee

Jubilarian: Wojciech Kryszewski

Book of Abstracts

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A Brezis-Nirenberg type result for a fourth order BVP

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In this lecture we discuss a Brezis-Nirenberg type result for a fourth order boundary value problem in the sub-critical regime. This result is motivated by the seminal and celebrated result due to Brezis and Nirenberg in [1].

We also discuss some recent results concerned with existence and regularity of solutions to a system of second order PDEs with Dirichlet boundary conditions. These aspects are a work in collaboration with Bernhard Ruf (Universita Statale di Milano) and Carlos Vélez (Universidad Nacional de Colombia Sede Medellín).

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Convexity properties of Yoshikawa-Sparr interpolation spaces

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One of the classical problems concerning interpolation theory consists in proving that the interpolation space obtained by a certain method of interpolation inherits properties of one of the spaces from the $(n+1)$-tuple of spaces. First result in this direction was obtained by B. Beauzamy who proved that if one of the spaces from the couple of spaces is uniformly convex then the interpolation space obtained by the Lions-Peetre method with a specific norm also has this property.

In this talk we present a similar theorem for Yoshikawa-Sparr interpolation method, which is a generalization of Lions-Peetre method, however we consider different norms in this space. We also present a method which enables us to obtain results concerning stability of infinite-dimensional counterparts of uniform convexity for the continuous version of Yoshikawa-Sparr interpolation method. In this method the Lebesgue-Bochner space $L_p(X)$ is used, so when we deal with such property as nearly uniform convexity or property (β), the difficulty is that $L_p(X)$ need not have this property even if the space X has it.

However our method enables us to obtain theorems for the continuous method in the easy way, using known results for the discrete one. We also present a new theorem concerning property (β) of a direct sum of Banach spaces, which we use to prove a theorem about interpolation.

The results presented in this talk are contained in the paper [1]

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Persistent Recurrence in Unimodal Maps

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We investigate the notion of persistent recurrence in unimodal maps. We use combinatoric tools such as kneading maps and Hofbauer towers to better understand how persistent recurrence relates to other dynamical properties of the map. We additionally aim to provide a symbolic characterization of when the turning point of the unimodal map is persistently recurrent.

Topological entropy of impulsive differential inclusions

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We will consider differential inclusions with nonautonomous multivalued impulses on tori. By a *topological entropy* of such systems, we mean the one of the compositions of the associated Poincaré translation operators along the trajectories with admissible (in the sense of Lech Górniewicz) impulsive maps. Although topological entropy is not a homotopy invariant, effective sufficient conditions of a positive entropy can be given just in terms of the Lefschetz numbers of given impulsive maps. This is possible due to the application of the generalized Ivanov-type inequality which allows us to estimate the entropy values from below by means of the asymptotic Nielsen numbers, jointly with the Anosov-type equality.

Our result generalizes and improves earlier theorems in this field, presented in the papers [1, 2, 3].

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Single orbits and the Wiener-Wintner theorem

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A "single orbit" approach to dynamics is the study of the interplay between the global properties of a dynamical system and the behaviour of its orbits. During the talk, I will discuss how much information about the system can be deduced from the existence of an orbit satisfying the conclusion of the Wiener-Wintner theorem (Wiener-Wintner generic orbit). I will present an Oxtoby type criterion for being a Wiener-Wintner generic point and will use it to characterise the spectrum of an invariant measure generated by such a point.

The talk is based on joint work with Melih Emin Can, Dominik Kwietniak and Piotr Oprocha.

Prescribed mass in the Born-Infeld problem and in a class of $(2, q)$-Laplacian equation

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In this talk, we will disclose the main results contained in a recent paper written jointly with Jarosław Mederski (Institute of Mathematics Polish Academy of Sciences) and Alessio Pomponio (Polytechnic University of Bari).

The importance of the electromagnetic theory announced by Born and Infeld, which is a nonlinear alternative to the classical Maxwell theory, lies in giving a unitarian point of view to describe electrodynamics and notable feature to be a fine answer to the well-known *infinite-energy problem*. Motivated by the fact that physicists are often interested in normalized solutions, we will discuss some recent results concerning existence and nonexistence of normalized solutions to a large class of quasilinear problems, including the Born-Infeld operator.

Our main theorems cover the mass subcritical, critical, and supercritical cases, in the sense of the critical exponents $2(1 + 2/N)$, $q(1 + 2/N)$. In the mass subcritical cases, we study a global minimization problem and obtain a ground state solution for a $(2, q)$-type operator which implies the existence of solutions to the Born-Infeld problem. We also deal with the mass supercritical cases, getting an existence result by a mountain pass approach, while in the critical cases, we prove nonexistence results by using asymptotic decays of particular externals.

Topological aspects of the Hopf bifurcation for discrete dynamical systems

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Let $f_\lambda : M \longrightarrow M$ with $\lambda \in [0,1]$ be a parametrized family of homeomorphisms of a manifold M. We say that an attractor $K \subseteq M$ of f_0 undergoes a *Hopf bifurcation* at $\lambda = 0$ provided that K is a repeller for f_λ for every $\lambda > 0$. Whenever an attractor undergoes a Hopf bifurcation, there appears a family of attractors K_λ that converges to K upper semicontinously as $\lambda \to 0$. In this talk we shall see that in many interesting situations we can characterize the Borsuk homotopy type of these attractors. These results have been obtained in collaboration with J.M.R. Sanjurjo.

The realization problem of attractors for knotted toroidal sets

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A *toroidal set* is a compactum $K \subseteq \mathbb{R}^3$ that possesses a basis of neighborhoods comprised of solid tori and is not cellular. It is said to be *unknotted* whenever it has a basis comprised of unknotted tori. Otherwise K is said to be a *knotted toroidal set*. In this talk we shall present necessary an sufficient conditions for a knotted toroidal set to be realized as an attractor for a homeomorphism of $\mathbb{R}^3$. We shall also see how to construct uncontably many toroidal sets that cannot be realized as attractors in both the knotted and unknotted cases. These results have been obtained in collaboration with J.J. Sánchez-Gabites.

Some questions related to metric fixed point problems and contributions

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In this talk we will discuss some questions related to classical metric fixed point problems. Specifically, one of them asks whether the closed unit ball of any infinite dimensional Banach space fails the FPP for Hölder-Lipschitz maps with null minimal displacement. Several partial results and open questions will also be displayed. Another issue we will discuss concerns the problem of obtaining sufficient conditions that guarantee weak FPP for a given class of maps. The content of the talk is related to some recent works by the author and in collaboration with co-authors, including published works [1], preprints [2] and preliminary research projects [3, 4].

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All but one expanding Lorenz maps with slope greater than or equal to $\sqrt{2}$ are leo

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We prove that, with only one exception, all expanding Lorenz maps with slope greater than or equal to $\sqrt{2}$ are locally eventually onto (leo). To be more precise, recall that an *expanding Lorenz map* is a map $f\colon [0,1] \to [0,1]$ satisfying the following three conditions:

1. there is a critical point $c \in (0,1)$ such that f is continuous and strictly increasing on $[0,c)$ and $[c,1]$,
2. $\lim_{x\to c^-} f(x) = 1$ and $\lim_{x\to c^+} f(x) = f(c) = 0$,
3. f is differentiable for all points not belonging to a finite set $F \subset [0,1]$ and there is $\lambda > 1$ such that $\inf\{f'(x) \mid x \in [0,1] \setminus F\} \geq \lambda$.

Recall also that f is called *locally eventually onto* (*leo* for short) if for every nonempty open set $U \subset [0,1]$ there is $n \in \mathbb{N}$ such that $[0,1] \setminus f^n(U)$ is finite. Assume that f is an expanding Lorenz map and $\beta = \inf\{f'(x) \mid x \in [0,1] \setminus F\}$. Let $f_0(x) = \sqrt{2}x + \frac{2-\sqrt{2}}{2} \pmod 1$. Our main result states that if $\beta \geq \sqrt{2}$ and $f \neq f_0$ then for every nonempty open subinterval $J \subset (0,1)$ there exists $n \in \mathbb{N}$ such that $f^n(J) \supset [0,1)$. In particular, f is leo. This is joint work with Piotr Nowak-Przygodzki.

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Critical point theory at infinity: an abstract approach and an application

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In this talk we present an abstract framework that allows to apply Morse theoretical methods to variational problems not satisfying the Palais-Smale condition. As an application we consider the Nirenberg problem of prescribing the scalar curvature on spheres, recovering and refining a result by Bahri-Coron, for instance.

The talk is based on joint work with Mohameden Ahmedou (Giessen).

Global bifurcation results for a delay differential system representing a chemostat model

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In this talk, we present a global bifurcation result for periodic solutions of the following delayed first order system, depending on a real parameter $\lambda \geq 0$,

$$\begin{cases} s'(t) = Ds^0(t) - Ds(t) - \dfrac{\lambda}{\gamma}\,\mu(s(t))x(t) & t \geq 0 \\ x'(t) = x(t)\big[\lambda\mu(s(t-\tau)) - D\big] & t \geq 0, \end{cases} \tag{1}$$

in which the following conditions hold:

(a) $s^0 : \mathbb{R} \to \mathbb{R}$ is continuous, positive and ω-periodic, where $\omega > 0$ is given,

(b) $\mu : [0, +\infty) \to [0, +\infty)$ is C^2 and verifies $\mu(0) = 0$ and $\mu'(s) > 0$, for any $s \in [0, +\infty)$,

(c) D, γ and the delay τ are positive constants,

System (1) has been studied in [1] and it represents a chemostat model, with a delay. The chemostat is a continuous bioreactor with a constant volume, in which one or more microbial species are cultivated in a liquid medium containing a set of resources with, in particular, a specific nutrient. The maps $s(t)$ and $x(t)$ are, respectively, the densities of the nutrient and of the microbial species at time t. The device receives continuously an input of liquid volume, described by $s^0(t)$, containing a variable concentration of the specific nutrient. It expulses continuously towards the exterior an output of liquid volume

containing a mixing of microbial biomass and nutrient. The model described by the system (1) assumes that the consumption of the nutrient has no immediate effects on the microbial growth, but we have a time interval $[t-\tau, t]$ in which the microbial species metabolize(s) the nutrient.

If (s, x) is any solution of (1) such that x vanishes at some t_0, then x turns out to be identically zero. Thus, the first equation in system (1) becomes linear and has a unique ω-periodic solution, which is positive and can be written as

$$v^*(t) = \int_{-\infty}^{t} e^{-D(t-r)} D s^0(r)\, dr.$$

For a sake of simplicity, assume that $\frac{1}{\omega}\int_0^\omega \mu(v^*(t))\, dt = D$.

In [1], the authors prove that

(a) *if $\lambda < 1$ (resp. $\lambda > 1$) and (s, x) is an ω-periodic solution, different from $(v^*, 0)$, then, $x(t) < 0$ (resp. $x(t) > 0$) for all $t \in \mathbb{R}$;*

(b) *if $\lambda = 1$, no ω-periodic solution is different from $(v^*, 0)$.*

Hence, it is quite natural to ask if $(v^*, 0)$ is a bifurcation point for ω-periodic solutions of (1) as well as to investigate the global behaviour of the bifurcating branches of such solutions. Here, we call *ω-triple* an element (λ, s, x) in which (s, x) is an ω-periodic solution of (1) corresponding to λ. Denote by E the Banach space $E := \mathbb{R} \times C^1_\omega \times C^1_\omega$, where

$$C^1_\omega = \{u \in C^1([0, \omega], \mathbb{R}) : u(0) = u(\omega) \text{ and } u'(0) = u'(\omega)\}.$$

Our main result is the following:

There exist in E exactly two connected components $\mathcal{C}_+$ and $\mathcal{C}_-$ of nontrivial ω-triples, which are unbounded, contain $(1, v^*, 0)$ in their closure and are such that every $(\lambda, s, x) \in \mathcal{C}_+$ verifies $\lambda > 1$, $0 < s < v^*$ and $x > 0$, while every $(\lambda, s, x) \in \mathcal{C}_-$ verifies $\lambda < 1$, $s > v^*$ and $x < 0$.

The proof uses, among other tools, the Crandall-Rabinowitz local bifurcation theorem [3] and a concept of degree introduced in [2] for Fredholm maps of index zero between Banach spaces.

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Normalized ground states of the nonlinear Schrödinger equation with at least mass critical growth

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We propose a simple minimization method to show the existence of least energy solutions to the normalized problem

$$\begin{cases} -\Delta u + \lambda u = g(u) \quad \text{in } \mathbb{R}^N, \ N \geq 3, \\ u \in H^1(\mathbb{R}^N), \\ \int_{\mathbb{R}^N} |u|^2 \, dx = \rho > 0, \end{cases}$$

where ρ is prescribed and $(\lambda, u) \in \mathbb{R} \times H^1(\mathbb{R}^N)$ is to be determined. The new approach based on the direct minimization of the energy functional on the linear combination of Nehari and Pohozaev constraints intersected with the closed ball in $L^2(\mathbb{R}^N)$ of radius ρ is demonstrated, which allows to provide general growth assumptions imposed on g. We cover the most known physical examples and nonlinearities with growth considered in the literature so far as well as we admit the mass critical growth at 0.

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Global-in-time regular unique solutions to the 1d thermoelasticity and time-asymptotic

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In the talk, we shall say about the construction of unique regular solutions to the minimal nonlinear system of the 1d thermoelasticity. The obtained solution has a positive temperature. Our approach is based on an estimate using the Fisher information, which seems completely new in this context. It is combined with a recent temperature inequality and embedding inequality, which allows us to obtain a new energy estimate. The latter is used in a half-Galerkin procedure to yield global solutions. The uniqueness and further regularity of such solutions are obtained.

Moreover, we present the result concerning the time-asymptotic of solutions. Namely, it will be shown that the displacement converges to 0, and the temperature converges to a constant function.

Parabolic systems with cross-diffusion: global existence versus finite time blowup

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Two toy models, both consisting of parabolic systems with nonlinear cross-diffusion terms, obtained after a slight modification of the nonlinearity of the usual doubly parabolic Keller–Segel system

$$u_t = \Delta u - \nabla \cdot (u \nabla \varphi),$$

$$\tau \varphi_t = \Delta \varphi + u,$$

are studied. For these toy models, with the same structure of steady states as is for the nonlinear heat equation $u_t = \Delta u + u^2$, we establish that for data which are, in a suitable sense, smaller than the diffusion parameter τ in the equation for the chemoattractant, we obtain global solutions, and for data larger than τ, a finite time blowup. In this way, we check that our size condition for the global existence is sharp for large τ. Results are based on papers in collaboration with Grzegorz Karch, Dominika Pilarczyk, Hiroshi Wakui and in particular on [1].

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A classification of Hénon maps in the presence of strange attractors

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In my talk I shall present my work with Sonja Štimac from [1]-[2] on Hénon maps with strange attractors (Wang-Young parameters [6]-[7]). First I shall explain a construction, inspired by a work of Crovisier and Pujals on mildly dissipative diffeomorphisms of the plane [3]-[5], of conjugacy of these maps to the shift homeomorphisms on inverse limits of dendrites with dense set of branch points, and a characterization of orbits of critical points in terms of these inverse limits. Then I will explain how this leads to a classification of conjugacy classes of such maps in terms of a single sequence of 0s and 1s.

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Analyzing Multifiltrations Using Multiparameter Discrete Morse Theory

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Multifiltrations, or multiparameter filtrations, arise in various contexts, notably in topological data analysis. Essentially, a multifiltration of a space X captures its evolution with respect to possibly many parameters. As they vary, we track the evolution of X through the multipersistent homology induced by the filtration, which describes the homological changes in X. In many cases, a multifiltration can be represented as a nested sequence of sublevel sets of a vector function $f : X \to \mathbb{R}^k$, called a multifiltering function.

In this presentation, we show how to get a better grasp of such functions using multiparameter discrete Morse (`mdm`) theory, which is an extension of Morse-Forman theory to vector-valued functions. Notably, we see that a multifiltering function can be associated to a `mdm` function. Also, we see link the critical simplices of this `mdm` function to the concept of Pareto set of a (smooth) vector map. Finally, we present some practical applications in data analysis.

Continuum-wise hyperbolicity

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In this talk, I will introduce the classical example of P. Walters of the pseudo-Anosov diffeomorphism of the two-dimensional sphere, note several properties that are not very clear at first glance, and discuss a generalization of hyperbolicity called continuum-wise hyperbolicity that is present in this example. Then I will discuss some consequences of cw-hyperbolicity and recent advances in the classification of cw-hyperbolic surface homeomorphisms.

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On semilinear elliptic systems with superlinear boundary conditions

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We investigate a system of elliptic equations characterized by superlinear and subcritical boundary conditions with a bifurcation parameter. By employing a combination of rescaling techniques and degree theory, one can show the existence of a connected branch of positive solutions bifurcating from infinity as the parameter approaches zero if the nonlinearities have power type growth at infinity. Under additional conditions on the nonlinearities near zero, we discuss the existence of a global, connected branch of positive solutions bifurcating from the line of trivial solutions, with a unique bifurcation point from infinity when the bifurcation parameter is zero. We employ bifurcation theory, degree theory, and sub- and super-solution method to obtain our results.

Renormalization in Lorenz maps – Completely invariant sets and periodic orbits

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Lorenz maps are piecewise monotone interval maps with a single discontinuity. Such maps appear as Poincaré maps in geometric models of well known Lorenz attractor, but they also have important connections with number theory, fractal geometry and neuronal dynamics. The main objects considered in this talk will be renormalizations of expanding Lorenz maps, i.e. certain return maps of an original Lorenz map to smaller intervals around the discontinuity.

In 2011, Yiming Ding in his paper [1] claimed that there is a one-to-one correspondence between the renormalizations and proper completely invariant closed sets of an expanding Lorenz map. However, in this talk we will present examples showing that the relation between these objects is more delicate. Namely, it turns out that a composition of so-called trivial renormalizations may be a renormalization, which do not have a corresponding proper completely invariant set. Based on this observation we provide complementary information to the statements in [1], which results in a better insight into the structure of renormalizations in Lorenz maps. The talk will be based on a joint work with Piotr Oprocha [2].

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Surface diffeomorphisms and their chaotic dynamics

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The ergodic theory of differentiable dynamics is deeply understood under a uniformly hyperbolic assumption [1]. Since the end of the 90's, huge progress has been obtained in order to cover larger classes of non-uniformly hyperbolic systems. During this lecture, I will discuss an alternative approach, which we recently introduced [2] with J. Buzzi and O. Sarig and which allows to describe every smooth surface diffeomorphism with positive topological entropy. These systems admit finitely many ergodic measures of maximal entropy satisfying nice statistical properties, and behave as countable Markov shifts with a spectral gap.

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Between Maxwell and Born–Infeld

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We present some recent results on a model of electromagnetic theory obtained from a Lagrangian density $\mathcal{L}_q$, depending on the parameter q. For $q = 1$, $\mathcal{L}_q$ corresponds to the Born–Infeld Lagrangian density and, for $q = 2$, it restores the Maxwell one.

Schauder estimates at nearly linear growth

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Variational integrals at nearly linear growth appear in the theory of plasticity with logarithmic hardening, that is the borderline configuration between plasticity with power hardening and perfect plasticity. The related (very challenging) regularity theory for minima has been intensively developed over the last 25 years, see e.g. the work of Frehse & Seregin '99, Fuchs & Mingione '00, Bildhauer '03, Beck & Schmidt '13, Beck & Bulíček & Gmeineder '20, Di Marco & Marcellini '20, Gmeineder & Kristensen '22, De Filippis & Mingione '23. In this respect, we will discuss an intrinsic approach to the theory of Schauder for general nonautonomous functionals at nearly linear growth that covers the most common model examples in the literature.

From recent, joint work with Cristiana De Filippis (Parma), Peter Hästö (Helsinki), and Mirco Piccinini (Pisa).

Overhanging solitary waves in the Water Wave Problem

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In the classical Water Wave Problem, we construct new overhanging solitary waves by a procedure resembling desingularization of the gluing of constant mean curvature surfaces by tiny catenoid necks. The solutions here predicted have long been numerically detected. This is joint work with Juan Davila, Monica Musso, and Miles Wheeler.

Persistent homology of simplicial maps with Dowker Complexes

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Consider the sequence of simplicial maps :

$$K_1 \xrightarrow{f_1} K_2 \xrightarrow{f_2} K_3 \xrightarrow{f_3} \dots \xrightarrow{f_{n-1}} K_n. \tag{2}$$

Our goal is to compute the persistent homology of (2). If all f_i are inclusion maps, then (2) is a filtration. In this cases, there is an efficient algorithm to compute the persistence. In [1], the authors decomposed each simplicial maps into a composition of elementary simplicial maps, and they construct another sequence where all maps are inclusions. They prove that the persistence diagram of (2) can be derived from the persistent diagram of the new sequence.

We tackle this problem with a different point of view by using Dowker complexes. Consider a relation $R \subset X \times Y$, and define two abstract simplicial complexes called Dowker complexes [2]. A simplex $[x_1, x_2, \dots, x_n] \in K_R$ if and only if there exists $y \in Y$ such that $(x_i, y) \in R$ for all $i = 1, 2, \dots, n$. Similarly, a simplex $[y_1, y_2, \dots, y_n] \in L_R$ if and only if there exists $x \in X$ such that $(x, y_i) \in R$ for all $i = 1, 2, \dots, n$. By the Dowker's Theorem, $|K_R|$ and $|L_R|$ are homotopically equivalent.

First, we show that we can adapt the method of elementary simplicial maps to Dowker complexes. Second, we use the Dowker's Theorem to transform (2) to a new sequence. Let $(f, g) : R \to R'$ be a pair of maps such that if $(x, y) \in R$ then $(f(x), g(y)) \in R'$. The pair (f, g) induces two simplicial maps $K_g : K_R \to K_{R'}$, and $L_f : L_R \to L_{R'}$. We can decompose $(f, g) = (f, id) \circ (id, g)$ up to some relation $\bar{R}$. For a simplicial map $f : K_1 \to K_2$, we can define the relations R_1 and R_2 such that $K_1 = K_{R_1}$, $K_2 = K_{R_2}$, and $(f, f) : R_1 \to R_2$ is well defined. We obtain a new sequence

$$K_1 = K_{R_1} \hookrightarrow K_{\bar{R}} \xrightarrow{\varphi_1} L_{\bar{R}} \hookrightarrow L_{\bar{R}_2} \xrightarrow{\varphi_2} K_{R_2} = K_2 \tag{3}$$

where φ_i are the homotopy equivalence from the Dowker's Theorem. Similar result as in [1] can be obtained, namely, one can derive the persistent diagram of (2) from the persistent diagram of (3).

This is a work in progress and a joint work with Tomasz Kaczynski and Mateusz Przybylski.

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Superposition operators of mixed order and jumping nonlinearities

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We discuss some recent results concerning a superposition operator of the form

$$\int_{[0,1]} (-\Delta)^s u \, d\mu(s),$$

for a signed measure μ on the interval of fractional exponent $[0,1]$, joined to a nonlinearity whose term of homogeneity equal to one is "jumping", i.e. it may present different coefficients in front of the negative and positive parts.

The signed measure is supposed to possess a positive contribution coming from the higher exponents that overcomes its negative contribution (if any).

The problem taken into account is also of "critical" type, though in this case the critical exponent needs to be carefully selected in terms of the signed measure μ.

Topological Data Analysis and Dynamics

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In this talk, we will discuss how tools from topological data analysis can be used to understand dynamics, with an emphasis on those known from finite samples. We will explore some standard invariants, such as persistent homology, to analyze solutions of partial differential equations [1]. Additionally, we will delve into tools at the intersection of topology and statistic [2], demonstrating how they can be used to analyze dynamical signals known from finite observations.

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Spectral Analysis of Nonlinear Operators Old Problems, New Results

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At the beginning of this talk we return back to 1970s when the speaker was a student of S. Fučík and J. Nečas at the Faculty of Mathematics and Physics of Charles University in Prague. During these years topological and variational methods of nonlinear analysis became modern topics which were beginning to be incorporated in the curricula for master programs. Actually, during this period famous Mountain Pass Theorem, Landesman-Lazer Conditions and Rabinowitz Bifurcation Theorem were born and during the same period celebrated papers of Landesman and Lazer, Ambrosetti and Prodi and Ambrosetti and Rabinowitz became were highly cited. Among Czech mathematicians, Svatopluk Fučík was very excited by all these new developments, and since his age was close to that of ours (he was only 9 years older) he succeeded to contaminate us by his enthusiasm. In a short period of time he made a lot of substantial contributions which were published in his numerous papers and books. However, his premature death did not allow him to raise enough continuators of his research. The speaker had the priviledge to be one of few students who were impacted by his personality and this fact determined his future professional career. Particular attention of Fučík and his colleagues was focused on the spectral analysis of nonlinear operators and their contribution to this topic was collected in the monograph of Fučík, Nečas, Souček and Souček, published by Springer Lecture Notes Series in 1973. The ideas to investigate nonlinear problems using asymptotic methods lead to the notions of the Fučík spectrum as well as to the spectrum of the p-Laplacian. Both concepts in ODE setting were studied in speaker's Master Thesis and later also in his PhD Thesis. The fact that this topic was relevant for the future development of nonlinear analysis is illustrated by the vast literature dealing with both of these concepts. Large number of new methods were developed in order to solve several open problems in this field. The goal of this talk is to recall some of the *old problems* which remain open after more than 50 years. On the other hand, we want to present some *new results* which illustrate that nonlinear stucture of the problem might lead to some surprising and unexpected results.

Invariant measures of $\mathscr{B}$-free subshifts

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Sarnak [5] turned attention to $\mathscr{B}$-free systems. For $\mathscr{B} \subseteq \mathbb{N}$, the $\mathscr{B}$-free subshift is the orbit closure of the characteristic function of the set of $\mathscr{B}$-free integers. We show that many results about invariant measures, previously only known for the hereditary $\mathscr{B}$-free subshift [4, 1], have their analogues for $\mathscr{B}$-free subshift. In particular, we discuss the recent proof of a conjecture of Keller [3] about a description of such measures. The talk is based on the joint work with Joanna Kułaga-Przymus and Daniel Sell [2].

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Fixed point index for multivalued maps on finite spaces

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Quite recently, the Lefschetz fixed point theorem for acyclic-valued maps $F : X \multimap X$ which are susc or slsc has been proved for X being finite T_0 topological space, see [1]. The authors used the correspondence between finite spaces and posets due to Aleksandroff. In particular, the order complex $K(X)$ consisting of all chains in the poset is a simplicial complex weekly homotopically equivalent to X. Every continuous map $f : X \to X$ induces a simplicial map $K(f) : K(X) \to K(X)$. Similarly, a multivalued acyclic map $F : X \multimap X$ gives rise to a chain map $\varphi : C_*(K(X)) \to C_*(K(X))$. and the Lefschetz numbers of the maps are the same.

Thus we present the notion of a 'local' Lefschetz number on a chain level as in H. Hopf paper [3]. This was the main idea of the notion of index systems in [4], where all the standard properties of the fixed point index were proved. Therefore we obtain a 'local' Lefschetz fixed point theory.

Let us note that another local invariant - Conley index, has been defined recently in [2].

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Growth rate of the Reidemeister numbers and topological entropy

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This is joint work with Mateusz Slomiany

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We study the asymptotic behavior of the Reidemeister numbers of iterates of an endomorphism of a group. We find a connection between the asymptotic Reidemeister number and the topological entropy. We also study the growth rate of the total dimensions of symplectic Floer homologies of iterates of the symplectomorphism of a surface.

Periodic solutions of ϕ-Laplacian differential systems: continuation theorems, bound sets, and atypical bifurcation

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We present some recent results about the existence of periodic solutions of vector differential systems involving ϕ-Laplacian operators. We first review the corresponding theorems concerning the linear differential operator, then new continuation theorems are exploited to obtain existence results with "bound set" conditions. At last, we provide atypical bifurcation results in the sense of Prodi–Ambrosetti. All the results rely on topological degree theory. The talk is based on a recent collaborations with Prof. Pierluigi Benevieri (University of São Paulo, Brazil) and Prof. Fabio Zanolin (University of Udine, Italy).

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Control Systems and Differential Inclusions on Wasserstein spaces

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In recent times models arising in social sciences involving large number of agents are often stated on Wasserstein metric spaces of Borel probability measures. The aim of this talk is to demonstrate that some corner stone results of classical differential inclusions known in the Euclidean framework have their analogues in Wasserstein spaces. Such an extension of the theory of differential inclusions is crucial to study control systems on Wasserstein spaces. Indeed, it is well known that for optimal control of ODEs, the differential inclisions theory provides useful tools to investigate existence of optimal controls, necessary optimality conditions and Hamilton-Jacobi-Bellman equations. Same happens for Wasserstein spaces.

In particular, I will present necessary and sufficient conditions for the existence of solutions to state-constrained continuity inclusions from [2] building on a suitable notion of contingent cones in Wasserstein spaces leading to viability and invariance theorems. In a bit less general setting they were already applied in [4], [5] to investigate stability of controlled continuity equations and uniqueness of solutions to HJB equations.

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Concentration phenomena in some chemotaxis system with local sensing

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In this talk, we consider the following chemotaxis system with local sensing:

$$\begin{cases} u_t = \Delta(e^{-v}u), \\ \tau v_t = \Delta v - v + u, \end{cases}$$

in the two-dimensional setting, where $\tau \in \{0, 1\}$. This system was introduced to describe a chemotaxis movement taking account of the local sensing effect ([3]). This system is also a simplified one introduced in [4].

This system resembles the well-known Keller–Segel system. Under suitable setting, they share the same set of equilibria and have the same Lyapunov functional. However, while finite-time blowup solutions can be constructed for the Keller–Segel system, in our system solutions exist globally in time independently of the magnitude of mass and we observe "delayed blowup": infinite-time blowup solutions. More precisely, a critical-mass phenomenon was observed that with any sub-critical mass, the global solution is uniformly-in-time bounded while with certain super-critical mass, the global solution will blow up at time infinity ([1, 2]). In this talk, we will construct the infinite-time blowup solutions and discuss their behaviour. This talk is based on a joint work with Takasi Senba (Fukuoka University).

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On solvability of nonlinear equations driven by coercive operators

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Let E be a separable reflexive Banach space. Given an operator $A : E \to E^*$, a fixed element $b \in E^*$ we will consider the solvability of

$$A(u) = b \tag{4}$$

under the following types of assumptions:

a) A is weakly continuous, potential with a coercive potential;

b) A is continuous and coercive or else continuous, potential with a coercive potential;

c) A is continuous, potential and the following inequality is satisfied: there is a number $R > 0$ such that

$$\langle A(u) - b, u\rangle \geq 0 \text{ for } \|u\| = R.$$

In order to examine the solvability of (4) we will exploit the Weierstrass-Tonelli Theorem together with the Galerkin type approximation in case a) and techniques related to the existence of generalized solutions in case b). For the case c) we will utilize Galerkin approximations with nonlinear programming techniques pertaining to the Karush-Kuhn-Tucker Theorem.

Applications will be given to:
a) the fourth order elastic beam equation with rigidly fasten ends;
b) and c) the Dirichlet problem governed by the competing $(p, q)-$Laplacian.

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Existence of a periodic solution for superlinear second order ODEs

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We prove a necessary and sufficient condition for the existence of a T-periodic solution for the time-periodic second order differential equation $\ddot{x} + f(t, x) + p(t, x, \dot{x}) = 0$, where f grows superlinearly in x uniformly in time, while p is bounded. The method is based on a fixed-point theorem which uses the rotational properties of the dynamics.

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Global bifurcation of periodic solutions of S^1-symmetric autonomous Newtonian systems

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The aim of the talk is to study the phenomenon of global bifurcation of periodic solutions for autonomous Newtonian systems with additional symmetries of the potential.

It is known that for an autonomous systems the space of solutions can be considered with S^1-symmetries, given by shift in time. Therefore one can use tools of S^1-equivariant topology to study the global bifurcation. However, if the problem has additional symmetries, in some cases one needs more sophisticated tools to detect bifurcation. We consider the equation with S^1-symmetry of the potential. Using the degree theory for T^2-equivariant gradient maps, we obtain the new bifurcational results and generalize known theorems.

Optimal transport techniques in geometric problems

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In the 1980s, Kohn and Strang observed that in the study of some two-dimensional optimal desing problems in plasticity, one can simplify the problem of finding the stress tensor by replacing it with a scalar stress function in such a way that the stress tensor is the rotation by 90 degrees of the gradient of the stress function; then, one needs to consider a constrained minimisation problem for the stress function, with the length of its gradient bounded from above. In recent years, a similar link between the optimal transport problem and the least gradient problem was discovered. On convex domains in two dimensions, given a solution to the optimal transport problem for suitably chosen source and target measures, the gradient of the solution to the least gradient problem (i.e., minimisation of total variation for Dirichlet boundary data) turns out to be a rotation by 90 degrees of the so-called transport density. This provides a one-to-one correspondence of the data and solutions to the respective problems. This greatly increases the number of available tools in the study of the least gradient problem. In this talk we explore this correspondence, some of its generalisations and consequences.

Realizability theory for partially ordered sets

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In fixed point theory, a sequence (a_n) of non-negative integers is called *realizable* if there exists some space X and a map $T\colon X \to X$ with

$$a_n = \mathrm{Fix}(T^n) = \#\{x \in X \mid T^n x = x\}$$

for all $n \geq 1$.

It is not difficult to observe that a sequence (a_n) is realizable if and only if $b_n = \frac{1}{n}\sum_{d|n} \mu(\frac{n}{d}) a_d$ (the so-called Dold coefficient) is a non-negative integer for each $n \geq 1$, where μ denotes the classical Möbius function, cf. [2]. On the other hand, a formal and more abstract theory of Dold coefficients was developed in [1], where instead of the divisibility relation on the integers a partial order on a poset is considered. The aim of this talk is to generalize the notion of realizability to the setting of partially ordered sets and to identify some counterparts of the statements from [3] that are valid in the classical case.

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Dynamical decompositions of the torus

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We present recent work on decompositions of the 2-torus that are invariant under homeomorphisms with strong dynamical properties. We state open questions related to this work and give partial solutions. We also construct new examples of such decompositions.

Approach to compactness in normed spaces through seminorms

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We present a new attitude to the (pre-)compactness in normed spaces. We introduce the notion of an equinormed set using a suitable family of semi-norms on the given normed space, satisfying some natural conditions. Having some freedom of choice of the set of seminormes that is used to define equinormed sets, we may prove known compactness criteria (e.g. Arzelà-Ascoli theorem in spaces $C(X,\mathbb{R})$ or in sequence spaces l^p for $p \in [1,+\infty)$) as well as such that were unknown until recently (e.g. in the spaces $BV([0,1])$ of functions of bounded variation on compact interval or $Lip(X,\mathbb{R})$ of Lipshitz continuous functions).

Global compactness result and multiplicity of solutions for a class of critical exponent problems in the hyperbolic space

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Consider the problems of the type

$$-\Delta_{\mathbb{B}^N} u - \lambda u = a(x)|u|^{2^*-2}u + f(x) \quad \text{in } \mathbb{B}^N, \quad u \in H^1(\mathbb{B}^N),$$

where $\mathbb{B}^N$ denotes the ball model of the hyperbolic space of dimension $N \geq 4$, $2^* = \frac{2N}{N-2}$, $\frac{N(N-2)}{4} < \lambda < \frac{(N-1)^2}{4}$ and $f \in H^{-1}(\mathbb{B}^N)$ ($f \not\equiv 0$) is a non-negative functional in the dual space of $H^1(\mathbb{B}^N)$. The potential $a \in L^\infty(\mathbb{B}^N)$ is assumed to be strictly positive, such that $\lim_{d(x,0)\to\infty} a(x) = 1$, where $d(x,0)$ denotes the geodesic distance. For $f = 0$ and $a \equiv 1$, profile decomposition was studied by Bhakta and Sandeep in [1]. However, due to the presence of the potential $a(.)$, an extension of profile decomposition to the present set-up is highly nontrivial. It requires several delicate estimates and geometric arguments concerning the isometry group (Möbius group) of the hyperbolic space. The result we achieved generalizes the profile decomposition. Further, using the decomposition result,

we derived various energy estimates involving the interacting hyperbolic bubbles and hyperbolic bubbles with localized Aubin-Talenti bubbles. Finally, combining these estimates with topological and variational arguments, we established a multiplicity of positive solutions in the cases: $a \geq 1$ and $a < 1$ separately. The equation studied can be thought of as a variant of a scalar-field equation with a critical exponent in the hyperbolic space, although such a critical exponent problem in the Euclidean space $\mathbb{R}^N$ has only a trivial solution when $f \equiv 0$, $a(x) \equiv 1$ and $\lambda < 0$.

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Polychromatic Surface Plasmons in Kerr-Nonlinear Media

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We study Maxwell's equation in a one-dimensional geometry on the interface between two media with a nonlinear and non-local (in time) dependence of the electric displacement field $\mathcal{D}$ on the electric field $\mathcal{E}$ of the form

$$\begin{aligned}\mathcal{D}(x,y,t) = \epsilon_0 \mathcal{E}(x,y,t) + \epsilon_0 \int_{\mathbb{R}} \chi^{(1)}(x,t-s)\,\mathcal{E}(x,y,s)\mathrm{d}s \\ + \epsilon_0 \int_{\mathbb{R}^3} \chi^{(3)}(x,t-s_1,t-s_2,t-s_3) \\ ((\mathcal{E}(x,y,s_1)\cdot\mathcal{E}(x,y,s_2))\,\mathcal{E}(x,y,s_3))\,\mathrm{d}(s_1,s_2,s_3).\end{aligned}$$

The monochromatic ansatz

$$\mathcal{E}(x,t) = e^{-\mathrm{i}\omega t}E(x) + e^{\mathrm{i}\overline{\omega} t}\overline{E(x)}$$

lets us study an associated spectral problem, which is well understood [1]. In the nonlinear setting however, this ansatz produces also time dependences with higher frequencies. These cannot be compensated by the linear terms of the equation and are thus commonly neglected in the physics literature.

To treat the full nonlinear problem, we introduce a polychromatic ansatz for a solution as an infinite series over integer multiples of the frequency ω and show the existence of such solutions in a physically meaningful setting.

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Rogue Waves for a Generalized Semilinear Wave Equation

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We study the generalized semilinear wave equation

$$V(x)u_{tt} + d(t)M(x,\partial_x)u = |u|^{p-1}u \text{ for } (x,t) \in \mathbb{R}^N \times \mathbb{R}$$

where M is elliptic and d is a positive periodic step potential. Our goal is to construct solutions which are localized in space and time (rogue waves) by means of variational methods. We present our approach with its main difficulties and discuss suitable examples for M and d. This is joint work with Wolfgang Reichel (KIT).

Lower bounds for travelling wave speed in asymmetrically supported beam

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This talk follows the lecture of Hana Levá, i.e., it deals with the same PDE with jumping nonlinearity

$$u_{tt} + u_{xxxx} + au^+ - bu^- + g(u) = 1, \quad x \in \mathbb{R},\ t > 0,$$

describing an asymmetrically supported beam and specifies admissible values of the wave speed c for which the problem possesses a homoclinic travelling wave solution. We show that in contrast to previously studied problems modelling suspension bridges, the presence of the term with negative part of the solution in the equation results in restrictions of c. We provide the maximal wave speed range for which the existence of the travelling wave solution can be proved using the Mountain Pass Theorem. We also introduce its close connection with related Dirichlet and periodic problems and their Fučík spectra. Moreover, we present several analytical approximations of the main existence result with assumptions that are easy to verify.

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Partial Blow-up Phenomena in the $SU(3)$ Toda System on Riemann Surfaces

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This talk discusses the partial blow-up phenomena for the $SU(3)$ Toda system on compact Riemann surfaces with boundary. We consider the following coupled Liouville system with Neumann boundary conditions:

$$\begin{cases} -\Delta_g u_1 = 2\rho_1 \left(\frac{V_1 e^{u_1}}{\int_\Sigma V_1 e^{u_1}\, dv_g} - \frac{1}{|\Sigma|_g} \right) - \rho_2 \left(\frac{V_2 e^{u_2}}{\int_\Sigma V_2 e^{u_2}\, dv_g} - \frac{1}{|\Sigma|_g} \right) & \text{in } \Sigma \\ -\Delta_g u_2 = 2\rho_2 \left(\frac{V_2 e^{u_2}}{\int_\Sigma V_2 e^{u_2}\, dv_g} - \frac{1}{|\Sigma|_g} \right) - \rho_1 \left(\frac{V_1 e^{u_1}}{\int_\Sigma V_1 e^{u_1}\, dv_g} - \frac{1}{|\Sigma|_g} \right) & \text{in } \Sigma \\ \partial_{\nu_g} u_1 = \partial_{\nu_g} u_2 = 0 & \text{on } \partial\Sigma \end{cases},$$

where (Σ, g) is a compact Riemann surface with smooth boundary $\partial\Sigma$, ρ_i is non-negative parameter and the positive potential function V_i is smooth for $i = 1, 2$.

On a bounded domain the corresponding problem has been studied by D'Aprile, Pistoia, and Ruiz in [1]. On Riemann surface with boundary, We construct a family of blow-up solutions via the Lyapunov-Schmidt reduction and variational methods, where one component remains uniformly bounded from above, and the other exhibits partial blow-up at a prescribed number of points, both in the interior and on the boundary. This construction is based on a non-degeneracy hypothesis for singular mean field equations.

I will discuss constructing solutions for the Toda system with partial blow-ups and analyze the non-degeneracy hypothesis that is important to our approach.

This is a joint work with Prof. Dr. Thomas Bartsch and Prof. Dr. Mohameden Ahmedou.

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Birkhoff-Kellogg type results with applications

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We present some classical and recent results of Birkhoff-Kellogg type in cones. We illustrate their applicability in the context of ordinary, functional and partial differential equations subject to local, nonlocal and functional boundary conditions.

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A convergence result for mountain pass periodic solutions of perturbed Hamiltonian systems

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The lecture is based on the work [2], where we study second order Hamiltonian systems under small perturbations. We assume that the main term of the system has a mountain pass structure, but do not suppose any condition on the perturbation. We prove the existence of a periodic solution. Moreover, we show that periodic solutions of perturbed systems converge to periodic solutions of the unperturbed systems if the perturbation tends to zero. The assumption on the potential that guarantees the mountain pass geometry of the corresponding action functional is of independent interest as it is more general than those by Rabinowitz [3] and the authors [1].

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Characterization of compact sets by fixed-point theorems

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Every fixed-point theorem has two key players: the domain and the operator involved. Some requirements over the topology or geometry of the domain, along with some criteria over the continuity or metric properties of the acting operator, are known that may guarantee the existence of a fixed point.

During this talk, we will focus on domains which are closed convex subsets of a Banach space and we will look at an opposite scope: What can we say about the topological or geometrical features of a domain C when the existence of a fixed point is always guaranteed for some family of Lipschitz operators acting over C? Can we determine any features over the Banach space X in which the domain lives?

Given a topological or metric space C, we say that C has the fixed point property for a given class of mappings $\mathcal{A}$, if every map $T : C \to C$, with $T \in \mathcal{A}$, has a fixed point.

During this talk, we will try to cover a different point of view: Is it possible to determine some topological or geometrical features of the domain C, when it has the fixed point property for a certain class of Lipschitzian mappings?

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Discrete Morse theory for multi-filtrations

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Abstract

I will present my joint work with Allili and Brouillette aimed to provide an application-driven extension of the Morse-Forman theory to vector-valued functions. Using concepts of combinatorial topological dynamics studied in recent years, in addition to adapting the main definitions and results of Forman to this multidimensional setting, we establish a more general result regarding the sublevel sets of a multidimensional discrete Morse function and find a set of Morse inequalities specific to such functions.

Structural stability of global attractors for a gradient ODE with delay

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We study the following gradient ODE

$$x'(t) = f(x(t)) \quad \text{where} \quad f \in C^2(\mathbb{R}^d;\mathbb{R}^d) \quad \text{and} \quad f = \nabla F. \tag{5}$$

Assuming that this equation has a global attractor, we consider the problem perturbed by the delay term

$$x'(t) = f(x(t)) + \varepsilon \int_{-\infty}^{t} M(t-s)x(s)\,ds, \tag{6}$$

where $M : [0,\infty) \to \mathbb{R}^{d\times d}$ is a time dependent matrix whose norm decays to zero exponentially as $t \to \infty$. The main result says, that if in the system (5) the equilibria are hyperbolic and their stable and unstable manifolds intersect transversally, then the structure of heteroclinic connections is preserved in the infinite dimensional system (6) if ε is small. The main tool is the Dafermos transform. We prove that in the perturbed problem the additional infinite dimensional variable that comes from the presence of the delay term after the Dafermos transform is always locally stable. This allows us to demonstrate that the local stable and unstable manifolds of all the equilibria of (6) when ε is small are C^1 close, in appropriate sense, to the local stable and unstable manifolds of (5). Consequently, the structure of transversal intersections is preserved in the perturbed system.

Stability of equilibria to generalized Navier-Stokes-Fourier system

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In a given domain we consider a generalized Newtonian incompressible heat conducting fluid with prescribed nonuniform temperature on the boundary of the domain and with the no-slip boundary conditions for the velocity. We study stability of equilibria if no external body forces are applied to the fluid. In dependence on the growth of the constitutively determined part of the Cauchy stress we identify different classes of proper solutions that converge to the equilibrium exponentially in a suitable metric. Consequently, the equilibrium is nonlinearly stable and attracts all weak solutions from these classes. We also show that these classes of solutions are nonempty.

Tangential homoclinic points locus of the Lozi maps

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We consider the dynamics of the two-parameter Lozi family of planar homeomorphisms, more precisely, the relationship between the stable and unstable manifold of the hyperbolic fixed point X of that family in the first quadrant together with their intersections, homoclinic points. We present curves in the parameter space which represent the border of existence of homoclinic points for X and determine all possible homoclinic points in the border case. Within the determined border, we introduce a specific region in the parameter space for which the period-two orbit is attracting and there are no homoclinic points for X. In this region we show that the Lozi map has zero topological entropy, expanding the results in [1].

This is joint work with Michał Misiurewicz (IUPUI, Indianapolis) and Sonja Štimac (University of Zagreb, Croatia).

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Bôcher type theorem for gradient perturbed Lévy operators: supercritical and subcritical cases

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A classical Bôcher's theorem asserts that any positive harmonic function (with respect to the Laplacian) in the punctured unit ball can be expressed as a linear combination of the Green function for the unit ball and a positive function that is harmonic in the whole unit ball. This theorem expresses one of the fundamental results in the theory of isolated singularities and it can be viewed as a statement on the asymptotic behavior of positive harmonic functions near their isolated singularities. We present a generalization of this results to Lévy operators perturbed by spatially inhomogeneous gradient operator. We propose a probabilistic potential theory approach which allows us to achieve the main result without any restrictions on the Lévy operator. In particular our result covers gradient perturbed fractional Laplacians with any index of stability bounded between zero and two - the method is therefore applicable to subcritical and supercritical cases.

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On certain chemotaxis-consumption-growth systems

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During the talk we will consider a chemotaxis-consumption system with logistic growth and realistic boundary conditions. The impact of Robin or Dirichlet boundary condition imposed on the chemoattractant density will be shown. We will discuss recent results concerning both the stationary and evolutionary problem.

Topological rigidity in Dehn twist homotopic classes

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Every orientation preserving 2-torus homeomorphism with no periodic orbit is either homotopic to the identity or (conjugate to a homeomorphism homotopic) to a Dehn twist. It is well known that the identity homotopy class is very flexible and contains a very rich variety of different dynamics among periodic point free homeomorphisms.

On the other hand, in this talk we will see that Dehn twist homotopy classes are topologically much more rigid and we shall prove that every non-wandering periodic point free homeomorphism in such a homotopy class is semi-conjugate to an irrational circle rotation.

The Dirichlet problem with the competing (p, q)–Laplacian with unbounded weight

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Lest us consider the following boundary value problem

$$\begin{cases} -\operatorname{div}\left(g(u)|\nabla u|^{p-2}\nabla u\right) + \operatorname{div}\left(|\nabla u|^{q-2}\nabla u\right) = f(x, u, \nabla u) & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is bounded domain with Lipschitz boundary $\partial\Omega$, $p > q > 1$, $g\colon \mathbb{R} \to \mathbb{R}$ and $f\colon \Omega \times \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}$. We shall investigate the existence of generalized solutions of the above problem. To obtain this result, we will use some abstract principle, which relies on the Galerkin scheme.

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Asymptotic stability of kinks in the odd energy space

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In this talk I will first present a 10 years old result about the asymptotic stability of the kink in the classical ϕ^4 model under the assumption of oddness of the initial perturbations. I will explain how the problem can be decomposed into radiation and internal modes and how the components can be controlled through virial estimates. This result depends on some numerical approximations and its proof can be viewed as computer assisted. Recently, we were able to generalize the asymptotic stability result to one dimensional scalar field models with one internal mode. I will show how using the Darboux factorization of the linearized operator around the kink one can avoid numerical approximations.

Tangency and the Intermediate Value Theorem: some new findings, applications and memories

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I will focus on presenting and discussing various results, including several new findings, related to Bolzano's intermediate value theorem and its extensions. My aim is to offer a unified approach to these issues, consistently relying on the concept of tangency—a geometric property strongly supported by topological arguments that constitutes one of the central tools in nonlinear functional analysis. The legacy of Bolzano and his followers remains significant and vibrant, serving as one of the cornerstones of contemporary mathematics. Additionally, I will share a few personal remarks and memories concerning mathematics and the individuals I have been fortunate enough to meet.

Bistability in a one-dimensional model of a two-predators-one-prey population dynamics system

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We study the classical two-predators-one-prey model, introduced in [1]

$$\dot{y}_i = m_i \frac{s-\lambda_i}{s+a_i} y_i, \qquad i=1,2, \qquad \text{(predators)}$$
$$\dot{s} = \left(1 - s - \frac{y_1}{s+a_1} - \frac{y_2}{s+a_2}\right) s \qquad \text{(prey)}$$

with all constants a_i, λ_i and m_i being positive. It is shown in [2] that for a broad range of parameter values the considered system exhibits a strong contraction in the (y_1+y_2)-direction in which case its dynamics can be approximated by the one-dimensional map given by a one-dimensional bimodal map:

$$x_{n+1} = f(x_n) = b + x_n - \frac{k}{1+e^{x_n}}$$

The map f has a negative Schwartzian derivative and, therefore, the considered system could have at most two attracting orbits (periodic or chaotic). We find where short-period regimes can be observed (in particular, we prove that there might be only one period-2 solution).

We describe analytically the structure of bifurcations of the map. Taking this mechanism into account, one can easily detect parameter regions where cycles with arbitrary high periods or chaotic attractors with arbitrary high numbers of bands coexist pairwise.

The domains where two attractors coexist are found out numerically.

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Properties of strange attractors in certain piecewise hyperbolic families

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We will discuss the existence and properties of strange attractors in a certain family of piecewise smooth maps exhibiting hyperbolic properties which is modelled on the Lozi family. In particular, we will introduce a Lozi-like family (see [1, 5]), which can be thought of as a C^1 perturbation of piecewise affine border-collision normal forms (see [4]). We will discuss recent results based on the renormalization introduced in [3] regarding the maximality and existence of strange attractors in the said family. We will also present cases where perturbation techniques fail in transitioning results from one dimension to two dimensions.

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Existence and multiplicity of solutions for a concave-convex system of ODEs

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In this lecture we discuss the system of ODEs

$$\begin{cases} -u'' = \lambda v^r + v^p & \text{in } (0,1), \\ -v'' = u^q & \text{in } (0,1), \\ u > 0,\ v > 0 & \text{in } (0,1), \\ u(0) = u(1) = 0, \\ v(0) = v(1) = 0, \end{cases} \tag{7}$$

where $\lambda > 0$, $r \in (0,1)$, $p \in (1,\infty)$, and q is such that $qr < 1$.

System (7) has been recently studied in higher dimensions by dos Santos, and Agudelo et al., motivated by the results of Ambrosetti, Brezis and Cerami from 1994. We explore the one dimensional case, which has not been treated yet and which usually renders more precise information about the solutions.

We discuss some results related to regularity, existence, and multiplicity of solutions of (7). We also present some numerical experiments exploring multiplicity of solutions of (7). Part of the discussion focuses on the well-posedness of these numerical experiments, which are based upon the *Poincaré-Miranda* theorem and its geometric ideas in the context of the *shooting method*. To our knowledge, such numerical approach has been extensively used for single equations, but is lesser-known for systems of ODEs.

Controlling rapid oscillations in flow structure interactions

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Flow-structure interactions are ubiquitous in nature and in everyday life. Flow or fluid by interacting with structural elements can lead to oscillations, hence impacting stability or even safety.

In this lecture we shall describe mathematical models describing the phenomena, These are represented by a 3 D Euler Equation coupled to a **nonlinear** dynamic elasticity on a 2 D manifold. Strong boundary-type coupling at the interface between the two media is at the center of the analysis. This provides for a rich mathematical structure, opening the door to several unresolved problems in the area of nonlinear PDE's, dynamical systems, related harmonic analysis and differential geometry. One of the central aspects is a question of uniqueness and sensitivity of *weak* solutions [ie of finite energy]. To contend with the difficulty, new methods in nonlinear analysis based on compensated compactness and harmonic analysis have been developed [4, 3].

Part of this talk is based on recent work [1, 2] and also work completed while the author was a member of the MSRI program "Mathematical problem in fluid dynamics" at the University of California Berkeley during the Workshop on Mathematical Theory of Fluid Dynamics, Spring 2021, Summer 2023

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Existence and the Number of Travelling Wave Solutions of the Beam Equation with Jumping Nonlinearity

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We focus on the travelling wave solutions of the boundary value problem for fourth–order partial differential equation

$$u_{tt} + u_{xxxx} + au^+ - bu^- + g(u) = 1, \quad x \in \mathbb{R},\, t > 0, \tag{8}$$

where $a,\ b > 0$, $u^\pm = \max\{\pm u, 0\}$ and $g\,(1/a) = 0$. Such problems are usually used as a model of an asymmetrically supported bending beam or as a generalised model of a suspension bridge.

First, we deal with the existence of the solution under significantly weakened assumptions compared to those previously used in literature. Using variational methods, in particular Mountain Pass Theorem together with a nonzero weak convergence after a suitable translation, we show that there are infinitely many homoclinic travelling wave solutions of the equation (8) with arbitrary wave speed from the interval $\left(c^*, \sqrt[4]{4a}\right)$, where $c^* > \sqrt[4]{4b}$. This limitation on the lower bound is due to the term bu^- in the nonlinearity.

Further, we present a special form of classical solutions of (8) which help us answer some open questions about number of travelling waves with the same fixed wave speed from the interval mentioned above.

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Topological entropy of multivalued maps in compact Hausdorff spaces

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We explore various approaches to extend the concept of topological entropy to multivalued mappings. Our talk involves the extension of existing notions and a comparative analysis with novel ones proposed. We demonstrate the applicability of certain topological entropy concepts to arbitrary multivalued mappings defined on compact Hausdorff spaces, which may not necessarily be metric.

Floer and Rabinowitz-Floer homologies for the Non-Linear Dirac Equation

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In the past few years, there have been many construction of a Morse-Floer type homology related to non-linear equations involving the Dirac operator on compact spin manifolds. In this talk I will discuss the construction of an isomorphism between the Floer-Rabinowitz homology corresponding to the non-linear Dirac equation and the Floer homology on corresponding time-energy extended phase space. This construction of this isomorphism is done through a careful analysis of a moduli space of hybrid flow lines.

This work is in collaboration with Takeshi Isobe.

Isometries between subspaces of codimension n of the space $C([1, \omega n])$

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For any given vector $x \in B_{\ell_1}$ and up to isometry, there exists exactly one ℓ_1-predual space X such that the ℓ_1 standard basis is $\sigma(\ell_1, X)$-convergent to x. We will explore what happens if we consider spaces for which the ℓ_1 standard basis has more w^*-cluster points.

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Juliusz Schauder (1899–1943). On the 125th anniversary of his birth

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Juliusz Schauder (1899–1943) was a Polish mathematician and a memeber of the so-called Lvov School of Mathematics, however he was working in the years 1925–1939 in secondary schools. From 1928 he was an assistant at the Jan Kazimierz University in Lvov, firstly at the Banach Chair and then at the Steinhaus Chair, but he gave only commissioned lectures and seminars, as he did not have a permanent position. From 1940 he was a professor at the University of Ivan Franko in Lvov. He published papers on functional analysis, differential equations, calculus of variation and orthogonal series. In the years 1926–1937 he wrote over 30 scientific papers. The name of Schauder in mathematics is related to Schauder's fixed point theorem, Schauder basis, Schauder compactness theorem, Schauder *a priori* estimates and the Leray–Schauder principle.

Of course, it will be many photos. The talk is based on my publication [1].

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Multiplicity results for Hamiltonian systems with Neumann-type boundary conditions

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We prove some multiplicity results for Neumann-type boundary value problems associated with a Hamiltonian system. Such a system can be seen as the weak coupling of two systems, the first of which has some periodicity properties in the Hamiltonian function, the second one presenting the existence of a well-ordered pair of lower/upper solutions.

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Extremal parametric curve for concave-convex system via Nehari manifold

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In this study we show the existence and multiplicity result for a strongly coupled concave-convex system for an *optimal* choice of involved real parameters via Nehari manifold method. We have obtained the parametric region which is optimal in the sense that the constraint minimization idea based on Nehari manifold is no longer applicable if the parameters lie in the exterior of the extremal region. By applying a finer analysis of fibering maps, we have shown the existence of atleast two positive solutions for the parameters lying below and even above the parametric extremal curve, characterized variationally via nonlinear generalized Rayleigh Quotient. The main result of the work is complimented with the study of the problem for negative parameter values.

Realization of prescribed set of minimal periods for the Morse-Smale diffeomorphisms

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A given self-map $f\colon M \to M$ of a compact manifold determines the sequence $\{l_n = L(f^n)\}$, of the Lefschetz numbers of its iterations. We consider its dual sequence $\{a_n(f)\}_{n=1}^{\infty}$ given by the Möbius inversion formula $a_n(f) = \frac{1}{n}\sum_{d|n}\mu(d)L(f^{\frac{n}{d}})$. The set $\mathcal{A}(f) = \{n : a_n(f) \neq 0\}$ is called the set of algebraic periods. We show that for every finite subset of $\mathcal{A} \subset \mathbb{N}$ of natural numbers there exist an orientable surface S_{g} of genus g and Morse-Smale diffeomorphism f of this surface such that $\mathcal{A}(f) = \mathcal{A}$. For any map from this class, as well as for every transversal map homotopic to it, this implies the existence of points with minimal period equal to $n \in \mathcal{A}$, n-odd. We also show that in the case when f reverses the orientation, there are restrictions on $\mathcal{A}$. Our result provides a solution to questions existing in the literature. During this talk we outline the topological part of this project based on the Nielsen-Thurston classification theorem.

The algebraic part will be presented also at the conference.

Boundary value problems for ordinary differential systems with generalized variable exponents operators

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In recent years an increasing interest in more general operators generated by Musielak-Orlicz functions is under development since they provide, in principle, a unified treatment to deal with ordinary and partial differential equations with operators containing the p-Laplacian, the ϕ-Laplacian, operators with variable exponents and the double phase operators.

These kind of considerations lead to consider quasilinear problems containing operators of the type $(\mathcal{S}(t,u'))'$, and look for nonlinear systems of ordinary differential equations of the form

$$(\mathcal{S}(t,u'))' = f(t,u,u')$$

submitted to various two-point boundary conditions.

Our approach consists to work in C^1 spaces to obtain suitable abstract fixed point and continuation theorems, from which several applications are obtained, including problems of Liénard and Hartman type.

This is a joint work with M. Garcia-Huidobro, R. Manásevich and S. Tanaka.

Multiple normalized solutions to a system of nonlinear Schrödinger equations

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We present recent results concerning normalized solutions to a system of coupled nonlinear Schrödinger equations. The problem appears in different areas of mathematical physics, e.g. in the analysis of Bose-Einstein condensation or in nonlinear optics. By means of spectral results, the Cwikel-Lieb-Rozenblum theorem, the Morse index and new Liouville-type results we show the existence of multiple normalized solutions for sufficiently large coupling. The talk is based on joint work with Andrzej Szulkin.

Algebraic periods of surface homeomorphisms

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A self-map $f\colon M \to M$ of a compact manifold determines the sequence $\{L(f^n)\}$, $n \geq 1$, of the Lefschetz numbers of its iterations. We consider its dual sequence $\{a_n(f)\}_{n=1}^{\infty}$ given by the Möbius inversion formula. The set $\mathcal{AP}(f) = \{n : a_n(f) \neq 0\}$ is called the set of algebraic periods of f. During the talk we describe finite sets of algebraic periods of homeomorphisms of an orientable surface, especially of Morse–Smale diffeomorphisms.

The talk is based on the joint project with G. Graff, W. Marzantowicz and A. Myszkowski.

Weak* fixed point property in ℓ_1

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A typical problem in the study of fixed point property for nonexpansive mappings is to characterize the spaces enjoying this property within a specific class of Banach spaces (see, e.g., [6, 5]).

We recall that the space X^* is said to have the weak* fixed point property (briefly, w^*-FPP) if for every nonempty, convex, w^*-compact subset C of X^*, every nonexpansive mapping (i.e., a mapping $T : C \to C$ such that $\|T(x) - T(y)\| \leq \|x - y\|$ for all $x, y \in C$) has a fixed point.

In this talk we study the w^*-fixed point property for the space ℓ_1 endowed with the weak* topologies generated by different preduals X. First, we provide some sufficient conditions for w^*-FPP in ℓ_1 based on the presence of particular subspaces in the predual X of ℓ_1. Then, we completely characterize w^*-FPP in ℓ_1 in terms of the existence of specific quotients of the predual space X. A key tool of our results is a detailed study of the hyperplanes of the space c of convergent sequences ([1]). Moreover, also a particular class of ℓ_1-preduals, the spaces of affine functions on Choquet simplex, plays an important role. Finally, we show that, in our characterizations, the existence of suitable quotients in the preduals X of ℓ_1 cannot be replaced by that of subspaces in X.

This talk is based on a series of papers written jointly with Emanuele Casini and Łukasz Piasecki ([2, 3, 4]).

The speaker is partially supported by INdAM - GNAMPA Project", codice CUP E53C23001670001.

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Critical $N-$Laplacian Stein-Weiss system in $\mathbb{R}^N$: Existence of solutions, asymptotic behavior, uniform estimates and regularity

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In this talk, we present a class of Stein-Weiss coupled system in dimension $N \geq 2$ involving Δ_N- operator, where $\Delta_N u = \mathrm{div}(|\nabla u|^{N-2}\nabla u)$ is the N-Laplacian operator. We discuss both linear and nonlinear coupling cases. Assuming that the nonlinearities have critical exponential growth in the sense of Trudinger-Moser inequality, we study the existence of positive solutions for the coupled system. Moreover, uniform estimates, asymptotic behavior and regularity of the solutions enrich the study of the system. In our approach we introduce an alternative to the standard arguments based on Lions' vanishing-nonvanishing and shifted sequences argument by utilizing a variant of Palais Principle of symmetric criticality.

Representations of Hamilton-Jacobi equations in optimal control theory for superlinear Hamiltonians

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The Hamilton-Jacobi equation

$$\begin{array}{c} -V_t + H(t,x,-V_x) = 0 \quad \text{in} \quad (0,T)\times\mathbb{R}^n, \\ V(T,x) = g(x) \quad \text{in} \quad \mathbb{R}^n, \end{array} \tag{9}$$

with a convex Hamiltonian H in the gradient variable can be studied with connection to optimal control problems. It is possible, provided that there exists a sufficiently regular triple (A, f, l) satisfying the following equality

$$H(t,x,p) = \sup_{a\in A}\{\,\langle\, p\,, f(t,x,a)\,\rangle\, -\, l(t,x,a)\,\}. \tag{10}$$

Then the value function of the optimal control problem defined by

$$V(t_0,x_0) = \inf_{(x,a)(\cdot)\,\in\, S_f(t_0,x_0)} \left\{ g(x(T)) + \int_{t_0}^{T} l(t,x(t),a(t))\,dt \right\}$$

represents the equation (9), where $S_f(t_0,x_0)$ denotes the set of all trajectory-control pairs of the control system $\dot{x}(t) = f(t,x(t),a(t))$, with $a(t)\in A$, for a.e. $t\in[t_0,T]$ and $x(t_0)=x_0$.

The triple (A,f,l), which satisfies the equality (10), is called a representation of H. In the literature (see [1], [2], [3], [4]), one can find constructions of representations for sublinear Hamiltonians (e.g., $H(t,x,p)=\alpha(t)|x||p|$). We demonstrate the construction of representations for superlinear Hamiltonians (e.g., $H(t,x,p)=\alpha(t)|x|^2|p|^2$). It is known that these representations are constructed using theorems on Lipschitz parametrizations of Lipschitz set-valued maps. In our case, we parametrize the set-valued map $E(t,x) = \mathrm{epi}H^*(t,x,\cdot)$ which is derived from the epigraph of the Legendre-Fenchel conjugate of $H(t,x,\cdot)$. While for sublinear Hamiltonians, the set-valued maps are Lipschitz continuous in the Hausdorff sense, for superlinear Hamiltonians, they exhibit Lipschitz continuity in the Aubin sense. The lack of parametrization theorems for the latter case is addressed by introducing an appropriate theorem for parametrizing set-valued maps with Lipschitz continuity in the Aubin sense.

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Homological persistence via the lens of combinatorial Morse theory

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In 1998, in his seminal work on discrete Morse theory, R. Forman [4, 5] presented an alternative to classical, ODE based, models in dynamics. Namely, he introduced and studied a purely combinatorial analogue of a vector field and the associated flow. In 2002 H. Edelsbrunner, D. Letscher and A. Zomorodian [2] introduced the concept of topological persistence which became the foundation of topological data analysis [6, 1]. In my talk, based on research in progress with H. Edelsbrunner [3] I will present some recently developed bridges between the two theories.

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Normalized solutions to a Choquard equation involving mixed local and nonlocal operators

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We study the existence of normalized solutions for the following Choquard type equation involving mixed diffusion type operators:

$$\begin{aligned} \mathcal{L}u + u &= \mu(I_\alpha * |u|^p)|u|^{p-2}u \text{ in } \mathbb{R}^n, \\ \int_{\mathbb{R}^n} |u|^2 dx &= \tau, \end{aligned}$$

where $\frac{n+\alpha}{n} \leq p \leq \frac{2s+n+\alpha}{n}$, $\tau > 0$ is a constant, $\mu > 0$ is a parameter, I_α is the Riesz potential of order $\alpha \in (0, n)$ defined by

$$I_\alpha = \frac{A_{n,\alpha}}{|x|^{n-\alpha}}, \text{ with } A_{n,\alpha} = \frac{\Gamma(\frac{n-\alpha}{2})}{\pi^{\frac{n}{2}} 2^\alpha \Gamma(\frac{\alpha}{2})} \text{ for every } x \in \mathbb{R}^n \setminus \{0\}$$

and the mixed operator $\mathcal{L}$ is given by

$$\mathcal{L} = -\Delta + \lambda(-\Delta)^s \text{ for some } s \in (0,1) \text{ and parameter } \lambda > 0.$$

We also provide regularity results of these solutions. Further, the equivalence between existence of normalized solutions and the existence of normalized ground states is established.

Variational approach for a nonlinear elliptic eigenvalue problem in Musielak-Orlicz spaces, concave - convex case

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We discuss the eigenvalue problem for a nonlinear elliptic equation of the form:

$$-Ax = -\lambda H_x(x) + Q_x(x) \tag{11}$$

where A is a nonlinear operator in a generalized modular space; H and Q are convex functionals. We derive a new variational methodology based on the Fenchel-Young conjugacy to prove the existence of solutions. Next, we apply the abstract result to a degenerate nonlinear elliptic eigenvalue problem in Musielak-Orlicz spaces.

Well-posedness Results in the Study of Quasi-Variational Inequalities

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We consider an elliptic variational inequality with unilateral constraints (QVI) in a Hilbert space X, specifying the assumptions under which a unique solution u is obtained. We formulate a convergence criterion for the solution u, outlining necessary and sufficient conditions on a sequence $\{u_n\} \subset X$, which guarantee the convergence $u_n \to u$ in the space X.

Next, we introduce a new well-posedness concept and show that it extends the classical Tykhonov and Levitin-Polyak well-posedness concepts for variational inequalities QVI. Finally, we present applications of our theoretical results in the study of a specific boundary value problem.

A talk is based on the resent results obtained together with Mircea Sofonea and Domingo A. Tarzia.

Interval maps with dense periodicity

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This talk is based on a recent study of the class of interval maps with dense set of periodic points CP and its closure Cl(CP) equipped with the metric of uniform convergence. We will focus on typical properties in the class of these maps, its geometric structure, conjugacy classes and related topics motivated by properties in topological dynamics and ergodic theory (e.g. entropy, mixing, etc.). We will compare our results with naturally related class of interval maps preserving Lebesgue measure.

The talk is based on joint works with J. Bobok, J. Činč and S. Troubetzkoy.

Existence and multiplicity of positive solutions for semipositone problems

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We discuss the existence and multiplicity of positive solutions for the following class of elliptic equations $\triangle u(x)+f(x,u(x))+g(x)x\cdot\nabla u(x)=0$, for $x\in\Omega_R=\{x\in\mathbb{R}^n,\|x\|>R\}$, $n>2$. Our goal is to show that the problem possesses nondecreasing sequences of solutions u satisfying the following condition: there exist $B>A>0$ and $L>0$ such that for all $x\in\mathbb{R}^n$, $\|x\|>L$, $A\|x\|^{2-n}\leq u(x)\leq B\|x\|^{2-n}$. We consider also the case when $f(x,\cdot)$ is negative at the origin, so-called semipositone problem. In the proof of these results we apply the subsolution and supersolution method developed by Noussair and Swanson.

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The Lyapunov exponent and rigorous computation of expansion in one-dimensional dynamics

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The Lyapunov exponent serves as a quantitative gauge of sensitivity to initial conditions within dynamical systems. A positive Lyapunov exponent is often associated with chaotic dynamics. In this study, we focus on one-dimensional dynamics and propose a methodology that is based on the analysis of weighted directed graphs [4] and uses interval arithmetic. Our approach aims to rigorously compute a lower bound for the expansivity rate that is supposed to correspond to the Lyapunov exponent.

We introduce the developed algorithm extending previous approaches [[1], [2], [3]] and perform numerical experiments checking the impact of various parameters on the results of the method, including computations on intervals of parameters, providing insights into its strengths and weaknesses. Additionally, we conduct a comparative analysis against a non-rigorous numerical approximation of the Lyapunov exponent.

Specifically, we evaluate our approach against a well-known family of quadratic maps, $f_a(x) = a - x^2$. This analysis not only evaluates the precision and validity of our computational framework but also sheds light on the limitations of a rigorous interval-based approach in contrast to non-rigorous numerical simulations when estimating the Lyapunov exponent.

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Positive solutions to nonhomogeneous quasilinear problems with singular and supercritical nonlinearities

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In recent years, there has been a growing interest in nonlinear singular elliptic PDEs. We study the existence of nonnegative solutions for the following quasilinear and singular elliptic problems with supercritical nonlinearity

$$\begin{cases} -\Delta_p u - \Delta_q u = \lambda \dfrac{h(x)}{u^\gamma} + u^\theta, \ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{12}$$

where Ω is an open, bounded subset of $\mathbb{R}^N (N \geq 3)$ with C^2 boundary, h is a positive real-valued function, $1 < p < q < \infty$ and λ, θ, γ are positive parameters. Our motivation for this problem is taken from [1], where the authors considered the following problem

$$\begin{cases} -\operatorname{div}(M(x)\nabla u) = \lambda u^{-\gamma} + u^\theta, \ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

Our objective is to investigate problem (12), focusing on the impact of singular and supercritical nonlinearities on the right-hand side, alongside the nonhomogeneous operator. In particular, for supercritical cases, i.e., $\theta \geq q^* - 1$, we prove the existence of solutions in a weak sense. To demonstrate the existence of a weak solution, we utilize the method of sub and supersolution.

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Positive solutions for slightly subcritical elliptic problems

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We focus on semilinear elliptic equations involving sign-changing weight function and a nonlinearity of subcritical nature understood in a generalized sense. Using an Orlicz-Sobolev space setting, we consider superlinear nonlinearities which do not have a polynomial growth, and state sufficient conditions guaranteeing the Palais-Smale condition. We study the existence of a bifurcated branch of classical positive solutions, containing a turning point, and providing multiplicity of solutions.

This is a joint work with Mabel Cuesta, ULCO, see [1].

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Stability of the weak* fixed point property in ℓ_1

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A nonempty, bounded, closed and convex subset C of a Banach space X has the *fixed point property* (briefly, FPP) if each nonexpansive mapping $T : C \to C$ (i.e., $\|T(x) - T(y)\| \leq \|x - y\|$ for all $x, y \in C$) has a fixed point. A dual space X^* is said to have the $\sigma(X^*, X)$-*fixed point property* (briefly, $\sigma(X^*, X)$-FPP) if every nonempty, convex, $\sigma(X^*, X)$-compact set $C \subset X^*$ has the FPP. We say that X^* enjoys the *stable* $\sigma(X^*, X)$-FPP if there exists $\gamma > 1$ such that Y^* has the $\sigma(Y^*, Y)$-FPP whenever the Banach–Mazur distance $d(X, Y) < \gamma$.

We give several characterizations of all preduals X of ℓ_1 such that X^* has the stable $\sigma(\ell_1, X)$-FPP.

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Topological-numerical analysis of recurrence in applied dynamical systems

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A topological method based on rigorous numerics and a set-oriented approach for the analysis of applied dynamical systems will be introduced. Using a finite representation of a dynamical system with respect to a cubical grid in the euclidean space, the purpose of the method is to split the dynamics into recurrent and gradient-like components, and to use the Conley index as well as direct recurrence analysis in order to provide insight into the structure of isolated invariant sets found in the system. The method also allows one to determine changes in the dynamics that occur when parameters of the system are varied. The method originates from [1] and has been considerably improved since then. An application of this method to the analysis of a two-dimensional discrete-time model of a neuron [2] will be discussed.

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Free boundary problems: Liouville equation and Bose-Einstein condensates

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The first result claims the existence of solutions with infinite mass for the Liouville equation with Dirichlet boundary conditions in a two dimensional doubly connected domain. The key ingredient in the construction is the solution of a suitable free boundary problem. The method of the proof inspired the second result which states the existence of a solution of a two component system of coupled non linear Schrödinger equations modeling the phase separation in the binary mixture of Bose–Einstein condensates. The results have been obtained in collaboration with Michał Kowalczyk and Giusi Vaira in [1, 2]

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Evans function, parity and nonautonomous bifurcation

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This is a joint work with Iacopo Longo (Imperial College, London, UK).

We provide an approachable and yet flexible sufficient condition for the bifurcation of bounded entire solutions to nonautonomous ordinary differential equations. This requires to relate the parity [1], which is a crucial tool in the abstract bifurcation theory of nonlinear Fredholm operators to the Evans function [3], an established concept for the stability analysis of traveling waves to evolutionary differential equations.

We illustrate that isolated zeros of the Evans function imply that critical spectral intervals of the Sacker-Sell (dichotomy) spectrum split, while sign changes of the Evans function are sufficient for local and global bifurcations of whole continua of bounded entire solutions.

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Schrödinger equation in dimension two with competing logarithmic self-interaction

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The aim of this talk is to deal with a scalar field equation in dimension 2 in presence of two logarithmic nonlocal terms in competition, namely

$$-\Delta u + (\log|\cdot| * |u|^2)u = (\log|\cdot| * |u|^q)|u|^{q-2}u, \qquad \text{in } \mathbb{R}^2, \tag{$\mathcal{P}$}$$

with $q > 2$.

Formally, solutions of ($\mathcal{P}$) can be found as critical points of the functional

$$I(u) = \int_{\mathbb{R}^2} |\nabla u|^2\,dx + \frac{1}{2}\int_{\mathbb{R}^2}\int_{\mathbb{R}^2} \log(|x-y|)|u(x)|^2|u(y)|^2\,dx\,dy - \frac{1}{q}\int_{\mathbb{R}^2}\int_{\mathbb{R}^2} \log(|x-y|)|u(x)|^q|u(y)|^q\,dx\,dy.$$

However, both the two nonlocal terms have not a fixed sign, are not well defined in $H^1(\mathbb{R}^2)$ and are in competition. This requires, as first step, a careful study of suitable weighted Sobolev spaces, with coercive potentials, and their embedding properties.

The results have been obtained in joint works with A. Azzollini, P. d'Avenia, S. Secchi.

A complete invariant for shift equivalence for Boolean matrices and finite relations

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Let X be a finite set. We can describe a relation R on X by a Boolean matrix, and conversely, a Boolean matrix yields a relation on X. In both settings, shift equivalence is a natural and important dynamical equivalence relation. It is strictly weaker than conjugacy, and corresponds roughly to "eventual conjugacy" ([1]). For matrices, one use is in the classification of shifts of finite type. For relations, it arises in defining a Conley index for computational approximations of dynamical systems. Classifications of shift equivalence are given in [2, 3, 4]. In this talk, I will present a complete invariant in terms of the period, the induced partial order on recurrent components, and the cohomology class of the relation on those components.

The result is as follows. There exists a least integer $p > 1$ such that there exists an integer $N > 0$ such that $R^{n+p} = R^n$ for all $n \geq N$; we call p the period of R. Additionally, R induces a partial order $R_{\leq}$ on the strongly connected components, determined by which components map to which. Finally, a choice of representatives of the strongly connected components induces a co-cycle $\xi \colon R_{\leq} \to \mathcal{L}_p$, where $\mathcal{L}_p$ is the collection of non-empty subsets of $\mathbb{Z}/p\mathbb{Z}$; we denote the cohomology class of ξ by $[\xi]$. Now, for a finite relation R, the triple $(R_{\leq}, p, [\xi])$ is a complete invariant of shift equivalence.

This is a joint work with Ethan Akin, Marian Mrozek and Jim Wiseman.

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Mixing properties of a class of expanding Lorenz maps on the interval

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Suppose that $f : [0,1] \to [0,2]$ is a continuous strictly increasing function which is differentiable on $(0,1) \setminus F$ where F is a finite set, and assume that $\inf f' > 1$. Define $T_f x := f(x) - \lfloor f(x) \rfloor$, where $\lfloor y \rfloor$ is the largest integer smaller or equal to y. A map of this form is called an expanding Lorenz map. Conditions on f implying topological transitivity of T_f are investigated. Moreover, also topological mixing and some other variants of mixing properties will be considered.

Examples will show that the conditions on f are necessary to obtain these mixing properties. For the linear case $f(x) = \beta x + \alpha$ one obtains even better results. Here one obtains for example topological transitivity if $\beta \geq \sqrt[3]{2}$ and $0 \leq \alpha < \frac{1}{\beta^2+\beta}$. It is also topologically mixing unless $f(x) = \sqrt[3]{2}\, x + \frac{2-\sqrt[3]{4}}{2}$.

Time-Periodic Waves for Maxwell Equations with Nonlinear Polarization

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The Maxwell equations govern the propagation of electromagnetic waves in matter. In many cases the material properties do not change when an electromagnetic wave propagates through them. However, for a class of materials, the refractive changes in a nonlinear way in the presence of a sufficiently strong electric field **E**.

In this talk I will consider a model for a class of materials with nonlinear polarization properties. I will further consider special geometries where one can prove the existence of propagating time-periodic electromagnetic waves which are localized in directions orthogonal to the propagation direction. This problem leads to a quasilinear hyperbolic nonlinear partial differential equation for the electric field **E**. Solutions with the above properties (localized, time-periodic, propagating) will be found by a variational principle. Numerical simulations will also be shown.

This is joint work with Sebastian Ohrem (KIT).

Normalised solutions to a Schrödinger equation with potential

Matteo Rizzi
Justus Liebig University

This is a joint work with Thomas Bartsch (Justus Liebig University, Giessen, Germany), Riccardo Molle (University of Roma 2, Rome, Italy) and Gianmaria Verzini (Politecnico di Milano, Milano, Italy)

In [1] we consider a Schrödinger type equation of the form

$$-\Delta u + (\lambda + V(x))u = |u|^{p-2}u \tag{13}$$

in $\mathbb{R}^N$, with a non radial potential V under the mass constraint

$$\int_{\mathbb{R}^N} v^2 = \rho^2. \tag{14}$$

We provide some sufficient conditions about V for existence of solutions $(u, \lambda) \in H^1(\mathbb{R}^N) \times (0, \infty)$ for powers $2+\frac{4}{N} < p < \frac{2N}{N-2}$. The potential is allowed to have singularities. λ appears as a Lagrange multiplier, due to the mass constraint (14). The proof is variational, based on a min-max argument.

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On fractional Orlicz-Hardy inequalities

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We establish the weighted fractional Orlicz-Hardy inequalities for various Orlicz functions. Further, we identify the critical cases for each Orlicz function and prove the weighted fractional Orlicz-Hardy inequalities with logarithmic correction.

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Using an invariant knot of a flow to detect additional invariant structure

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Consider a continuous flow in $\mathbb{R}^3$ and suppose $N \subseteq \mathbb{R}^3$ is a compact 3–manifold such that the trajectories of the flow either cross ∂N transversally or bounce off it from the outside. Suppose we know that there exists an invariant knot or link K in the interior of N and want to look for additional invariant structure inside N. The following theorem holds: if K is contractible (in N) and nontrivial (in the sense of knot theory), then every neighbourhood U of K contains a point $p \in U \setminus K$ such that the trajectory through p is entirely contained in N. In other words, the presence of a contractible invariant knot in N forces the existence of additional invariant structure in N which, moreover, passes arbitrarily close to K.

The proof of this result makes use of a "coloured handle" theory which may be of independent interest to study flows in 3-manifolds. The goal of the talk is to introduce these tools and give an idea of how to prove the theorem using them.

Topology and dynamics of non-saddle sets

Jose Sanjurjo
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The theory of non-saddle sets can be considered as a general theory of stability and attraction, extending the classical one and encompassing recent developments such as the theory of unstable attractors without external explosions. The authors have previously studied some topological properties of non-saddle sets and their region of influence. In this talk we present three new results, the first of which establishes an exact sequence relating the cohomology of a global non-saddle set of a flow on a manifold to the cohomology of the manifold. The second one similarly establishes an exact sequence for the non-saddle decompositions of flows in manifolds and the third studies some bifurcation properties of non-saddle sets of flows in the plane. These results have been obtained in collaboration with Hector Barge and Jaime Sánchez-Gabites.

Title

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In this paper we are going to discuss compactness in Lorentz sequence spaces. Firstly, it will be shown how to define such a space, check whether a sequence belongs to it and calculate its norm. Equipped with this knowledge, we will proceed to propose usable compactness criteria for Lorentz sequence spaces, employing the concept of seminorms.

Normalised solutions to a fractional Schrödinger equation in the strongly sublinear regime

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Schrödinger-type equations model a lot of natural phenomena and their solutions have interesting and important properties: one of them is the conservation of mass, which gives rise to the search for *normalised solutions*. In this talk, I will explain a possible approach to solve

$$\begin{cases} (-\Delta)^s u + \mu u = g(u), \\ \int_{\mathbb{R}^N} u^2 \, \mathrm{d}x = m, \\ (\mu, u) \in \mathbb{R} \times H^s(\mathbb{R}^N), \end{cases}$$

where $N \geq 2$, $0 < s < 1$, and $m > 0$ is prescribed, in cases that include the so-called *strongly sublinear regime*, i.e. when

$$\lim_{t \to 0} \frac{g(t)}{t} = -\infty,$$

which makes the usual approach impossible because the energy functional is not well-defined over $H^s(\mathbb{R}^N)$. In the proposed approach, a family of approximating problems is considered so that the energy functional is of class $\mathcal{C}^1$ and a corresponding family of solutions is obtained, which eventually converge to a solution to the original problem.

This is joint work with Marco Gallo (Catholic University of the Sacred Heart, Brescia, Italy).

On nonlinear critical problems involving the Grushin Subelliptic Operator: bifurcation and multiplicity results

Simone Secchi
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I will present some recent results on a nonlinear PDE involving the Grushin operator with critical nonlinearity. This talk is based on a joint work with P. Malanchini (Milano Bicocca, Italy) and G. Molica Bisci (Urbino, Italy)

About entropy in B-free subshifts

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For a set $\mathcal{B} \subseteq \mathbb{N}$, let η denote the characteristic sequence of $\mathcal{B}$-free numbers. Its orbit closure is a 0-1-subshift X_η, called the $\mathcal{B}$-free subshift. I will present results from joint work with Aurelia Dymek and Joanna Kułaga-Przymus [2] about the entropy of X_η. A central object for this is the Toeplitz sequence η^* that generates the unique minimal component of X_η. If η^* is regular, we obtain results that are analogous to the ones already known in the hereditary case. Our main tool is, that for taut $\mathcal{B}$ the elements of X_η can be described as sequences "between η^* and η" by a result of Gerhard Keller [1].

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Testing topological conjugacy of time series

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We consider a problem of testing topological conjugacy of two trajectories coming from dynamical systems (X, f) and (Y, g). More precisely, given $x_1, ...x_n \subset X$ and $y_1, ..., y_n \subset Y$ such that $x_{i+1} = f(x_i)$ and $y_{i+1} = g(y_i)$ (for some unknown maps f and g), as well as $h : X \to Y$, we deliver a number of tests to check if the corresponding trajectories of f and g are topologically conjugated via h. The values of the tests are close to zero for systems conjugate by h and large for systems that are not. For our main developed method, ConjTest, the convergence of the test values, in case when sample size goes to infinity, is established.

We provide numerical examples indicating scalability and robustness of the presented methods. In addition, we show how the presented method gives rise to a test of sufficient embedding dimension, mentioned in Takens' embedding theorem. Finally, we include a proof-of-concept study using the presented methods to search for an approximation of the homeomorphism conjugating given systems.

The talk is based on a joint work with Paweł Dłotko (IM PAN, Dioscuri Centre in TDA) and Michał Lipiński (Institute of Science and Technology Austria).

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New compactness estimates for aggregation-diffusion equations

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I will present two results dealing with the passage to the limit in aggregation-diffusion equations where obtaining standard compactness estimates is difficult. The first result, obtained in collaboration with C. Elbar and B. Perthame, concerns the kinetic derivation of the degenerate Cahn-Hilliard equation from a certain nonlocal partial differential equation. The challenge here is that all necessary a priori estimates can only be obtained for the nonlocal quantities, providing almost no information about the limiting solution itself. We introduce a novel condition on the kernel that allows us to exploit the available nonlocal a priori estimates. The second result, obtained in collaboration with J. A. Carrillo and Y. Salmaniw, concerns the existence (and uniqueness) of solutions to aggregation-diffusion equations where the kernel is only bounded and integrable, for instance, a characteristic function of a ball or a cube. Here, we take advantage of the gradient flow structure in a novel way, utilizing the dissipation of free energy and equiintegrability to control the gradient of the solution. This second work is particularly important in ecology, where the case of a characteristic function of a cube is widely used as a toy model to study the dynamics of populations.

Wavefront for reaction-diffusion systems with degenerate diffusivity

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This talk explores the dynamics of systems governed by two coupled reaction-diffusion equations, wherein one component exhibits doubly degenerate diffusivity. These systems lack total mass conservation, separating them from standard scalar reaction-diffusion equations. We focus on searching for wavefront solutions, namely those that exhibit profiles with a constant speed of propagation characterized by a pair of strictly monotone functions. Through the application of shooting methods and fixed-point techniques, we establish conditions for the existence of these wavefronts and provide estimates for threshold speeds. Moreover, we discuss wavefront regularity and prove that all profiles are smooth except for the one at the threshold speed, which displays a distinct "sharp" behavior. Such models are frequently encountered in studying the spatial-temporal development of bacterial colonies on nutrient-rich agar plates.

The talk is based on joint works with L. Malaguti and V. Taddei (University of Modena and Reggio Emilia) and E. Muñoz-Hernández (Complutense University of Madrid).

On the existence of periodic orbits and chaotic dynamics

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We present two theorems on the existence of periodic solutions of autonomous differential equations inside isolating blocks. The first theorem was published in [1], where a combinatorial approach to the problem, based on a suitable decomposition of the phase space, was proposed. The second theorem is motivated by a corresponding result on time-periodic non-autonomous system given in [2]. It is illustrated by a proof of the existence of chaotic dynamics in systems imitating the behavior of the Lorenz system at some range of parameter values.

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The configuration space of at most n points on the circle

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We present some preliminary work on set-valued maps $f : X \multimap Y$ where the cardinality of $f(x)$ is at most n for every x. We discuss relations to several other theories, including Schirmer's theory of n-valued maps, Crabb's theory of "structured n-valued maps," symmetric product maps, and Skiba's "multivalued weighted maps". We also will discuss the topology of the configuration space $C_n(X)$ of at-most-n points in some space X. Specifically we describe the homology and fundamental groups of $C_n(X)$ when X is the interval, the circle, or $\mathbb{R}^n$. The most interesting example is the circle.

Tolman-Oppenheimer-Volkoff Equation

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We extend in [1] our results published in Comm. Math. Phys. 2021 [8] and J. Diff. Eqs 2023 [4] to cover relativistic case similarly as in Math. Meth. Appl. Sci. 2023 [3] modelling dark matter model for Tolman-Oppenheimer-Volkoff Equation

$$-rc^2p'(r)(rc^2-2Gm(r))=G(c^2m(r)+4\pi r^3p(r))(c^2\rho(r)+p(r))$$

as an alternative to black hole model studied recently by Klainerman, Szeftel and Giorgi [5] and Dafermos, Holzegel, Rodnianski and Taylor [6] both in static Schwarzschild and rotating Kerr geometries. For the introduction see the review papers of Giorgi and Bieri. The results obtained by Genzel and Ghez for Sagittarius A* were analyzed by Ruffini [7] and Chavanis [2] in the framework of dark matter with the modified relativistic Fermi-Dirac or Michie-King distribution function yielding the relevant equation of state providing the energy momentum tensor for Einstein equation. We analyze the dynamical system for which the global Lyapunov function is obtained thus yielding the limit mass for the system with gravitational collapse into a black hole.

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Multiplicity of solutions for Schrödinger equation with sign-changing nonlinearity

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We are going to show the existence of multiple solutions of Schrödinger equation

$$-\Delta u + V(x)u = f(u) - \lambda g(u) \tag{15}$$

where $u : \mathbb{R}^N \to \mathbb{R}$ and V is periodic with respect to $x \in \mathbb{R}^N$ (i.e. is symmetric under the action of $\mathbb{Z}^N$). This equation doesn't fit the assumptions of the famous paper [2], especially due to the change of the sign of the right-hand side.

The existence result for equation (15) was proven in [1]. We prove the theorem providing the existence of multiple critical points of the abstract functional in the way that the variational functional of equation (15) fits to it. During the talk we will show the idea of proving multiplicity results introduced in [2] and the modifications needed in this approach to cover a study of equation (15).

Join work in progress with Bartosz Bieganowski and Federico Bernini.

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Configuration spaces and multiple positive solutions to a singularly perturbed elliptic system

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We consider the elliptic system of ℓ equations

$$\begin{cases} -\varepsilon^2\Delta u_i + u_i = \Big(\sum\limits_{j=1}^{\ell} \beta_{ij}u_j^2\Big)u_i, \\ u_i \in H_0^1(\Omega), \quad u_i \neq 0, \quad i = 1, \dots, \ell, \end{cases} \tag{$*$}$$

where $\varepsilon > 0$ is a small parameter, Ω is a bounded smooth domain in $\mathbb{R}^N$, $N = 2$ or 3, $\ell \geq 2$, $\beta_{ii} > 0$ and $\beta_{ij} = \beta_{ji} < 0$ if $i \neq j$. The equation in ($*$) corresponding to $\ell = 1$ is $-\varepsilon^2\Delta u + u = \beta_{11}u^3$ and for certain Ω, e.g. Ω = ball, there exists exactly one positive solution. If $\ell = 2$, then the equations in ($*$) are

$$-\varepsilon^2\Delta u_i + u_i = (\beta_{ii}u_i^2 + \beta_{ij}u_j^2)u_i, \quad i = 1,\ j = 2 \text{ or } i = 2,\ j = 1,$$

and it is known that for small ε there exist at least 2 positive solutions which concentrate as $\varepsilon \to 0$. A natural question is whether the minimal number of positive solutions increases with ℓ. We show that this is the case and if ε is small, then the number of such solutions is at least ℓ. The proof is by estimating the Lusternik-Schnirelman category of a certain level set of the functional associated with ($*$) in terms of the category of suitable configuration spaces. A crucial fact is that the category of the configuration space of ℓ points in $\mathbb{R}^N$ equals ℓ (a result due to F. Roth).

This is joint work with Mónica Clapp and Alberto Saldaña.

Some results on domain variation of the first eigenvalue of the Laplacian

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We consider the first eigenvalue λ_1 of Laplacian with mixed boundary conditions on domains of the form $\Omega \setminus \mathscr{O}$. Under some geometric assumptions on Ω and $\mathscr{O}$, we prove the strict monotonicity of $\lambda_1(\Omega \setminus \mathscr{O})$ with respect to certain variations of $\mathscr{O}$ in Ω.

Multiplicity of nodal steady-states for classical logistic equations

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Using as a base the result on multiplicity of 1-node solutions for degenerate logistic equations by López-Gómez and Rabinowitz [1], I will first show how multiplicity also occurs for classical logistic equations, for weights which are small perturbations of the degenerate case. Then, I will show how multiplicity is valid far beyond this perturbative case, even arriving at cases arbitrarily close to situations where there is uniqueness.

These results are joint work with P. Cubillos and J. López-Gómez (Universidad Complutense de Madrid) [2].

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A priori regularity estimates for equations degenerating on nodal sets

Susanna Terracini

This talk is based on a joint works with Giorgio Tortone[2] and Stefano Vita[1]
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We prove *a priori* and *a posteriori* Hölder bounds and Schauder $C^{1,\alpha}$ estimates for continuous solutions to singular/degenerate equations with variable coefficients of type

$$\operatorname{div}(|u|^a A\nabla w) = 0 \qquad \text{in } \Omega \subset \mathbf{R^n},$$

where the weight u solves an elliptic equation of type $\operatorname{div}(A\nabla u) = 0$ with a Lipschitz-continuous and uniformly elliptic matrix A and has a nontrivial, possibly singular, nodal set.

Such estimates are uniform with respect to u in a class of normalized solutions having bounded Almgren's frequency. More precisely, we provide *a priori* Hölder bounds in any space dimension, and Schauder estimates when $n = 2$. When $a = 2$, the results apply to the ratios of two solutions to the same PDE sharing their zero sets. Then, one can infer higher order boundary Harnack principles on nodal domains by applying the Schauder estimates for solutions to the auxiliary degenerate equation. The results are based upon a fine blow-up argument, Liouville theorems and quasiconformal maps.

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This is a joint work with Věra Krajščáková (Palacký University in Olomouc, Czechia). We investigated a Dirichlet problem in one-dimensional billiard space

$$\begin{aligned} &x'' = f(t, x, x') \quad \text{if } x(t) \in \operatorname{int} K, \\ &x'(t+) = -x'(t-) \quad \text{if } x(t) \in \partial K, \\ &x(0) = A, \quad x(T) = B, \end{aligned}$$

where $T > 0$, $K = [0, R] \subset \mathbb{R}$, $R > 0$, f is a Carathéodory function on $[0, T] \times K \times \mathbb{R}$, $A, B \in \operatorname{int} K$. We found sufficient conditions for the existence of solutions having prescribed number of impacts with the boundary. Unlike the previous works [1, 2, 3, 4, 5], the right hand of the differential equation depends on the derivative of the solution. The results have been found especially for linear and sublinear growth of the right hand side in the last variable.

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Fixed point indices of iterates of orientation-reversing homeomorphisms

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Let $f\colon \mathbb{R}^m \to \mathbb{R}^m$ be a self-homeomorphism and let p be an isolated fixed point for each iterate of f. Therefore, the fixed point index, $\operatorname{ind}(f^n, p)$, is a well-defined integer for each n, and thus it is possible to consider a whole sequence of indices $\{\operatorname{ind}(f^n, p)\}_{n=1}^{\infty}$. It is known (cf. [2]) that under the assumption that $\{p\}$ is an isolated invariant set, the possible forms of indices in the class of orientation-reversing self-homeomorphisms of $\mathbb{R}^3$ are very restricted. In this talk, first, we present the complete solution to Problem 10.2 in [1], and secondly, rejecting the assumption that $\{p\}$ is necessarily an isolated invariant set, we give a full characterization of $\{\operatorname{ind}(f^n, p)\}_{n=1}^{\infty}$ in the class of orientation-reversing self-homeomorphisms of $\mathbb{R}^m$. This is a joint work with Prof. Graff.

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Semilinear exterior problems with critical nonlinearities

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The present work studies the global behavior of solutions to the exterior problem for the evolution equations with critical nonlinearities. For the considered problems, nonexistence results are obtained, which complements the interesting recent results by Ikeda et al. [J. Differential Equations, 269 (2020), no. 1, 563-594], where critical cases were left open. Moreover, our results provide partially answers to some other open questions previously posed by Zhang [Proc. Roy. Soc. Edinburgh Sect. A, 131 (2001), no. 2, 451-475] and Jleli-Samet [Nonlinear Anal., 178 (2019), 348-365].

Sheet happens (but only as the root of $1 - s$)

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We discuss the regularity properties of two-dimensional stable s-minimal surfaces, presenting a robust $C^{2,\alpha}$-estimate and an optimal sheet separation bound, according to which the distance between different connected components of the surface must be at least the square root of $1 - s$.

The extension of traces for Sobolev mappings between manifolds

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Given compact Riemannian manifolds $\mathcal{M}$ and $\mathcal{N}$ and $p \in (1, \infty)$, the question of traces for Sobolev mappings consists in characterising the mappings from $\partial\mathcal{M}$ to $\mathcal{N}$ that can arises of maps in the first-order Sobolev space $\dot{W}^{1,p}(\mathcal{M}, \mathcal{N})$. A direct application of Gagliardo's characterisation of traces for the linear spaces $\dot{W}^{1,p}(\mathcal{M}, \mathbb{R})$ shows that traces of maps in $\dot{W}^{1,p}(\mathcal{M}, \mathcal{N})$ should belong to the fractional Sobolev-Slobodeckiĭ space $\dot{W}^{1-1/p,p}(\partial\mathcal{M}, \mathcal{N})$. There is however no reason for Gagliardo's linear extension to satisfy the nonlinear constraint imposed by $\mathcal{N}$ on the target.

In the case $p > \dim \mathcal{M}$, Sobolev mappings are continuous and thus traces of Sobolev maps are the mappings of $\dot{W}^{1-1/p,p}(\partial\mathcal{M}, \mathcal{N})$ that are also restrictions of continuous functions [2]. The critical case $p = \dim \mathcal{M}$ can be treated similarly thanks to their vanishing mean oscillation property [2, 3, 6].

The case $1 < p < \dim \mathcal{M}$ is more delicate. It was first proved that when the first homotopy $\pi_1(\mathcal{N}), \dots, \pi_{\lfloor p-1 \rfloor}(\mathcal{N})$ are *trivial,* then the trace operator from $\dot{W}^{1,p}(\mathcal{M}, \mathcal{N})$ to $\dot{W}^{1-1/p,p}(\partial\mathcal{M}, \mathcal{N})$ is surjective [4]. On the other hand, several conditions for the surjectivity have been known: topological obstructions require $\pi_{\lfloor p-1 \rfloor}(\mathcal{N})$ to be *trivial* [2, 4] whereas analytical obstructions arise unless the groups $\pi_1(\mathcal{N}), \dots, \pi_{\lfloor p-1 \rfloor}(\mathcal{N})$ are *finite* [1] and, when $p \geq 2$ is an integer, $\pi_{p-1}(\mathcal{N})$ is *trivial* [5].

In a recent work, I have completed the characterisation of the cases where the trace is surjective, proving that the known necessary conditions turn out to be sufficient [7]. I extend the traces thanks to a new construction which works on the domain rather than in the image. When $p \geq \dim \mathcal{M}$ the same construction also provides a Sobolev extension with linear estimates for maps that have a continuous extension, provided that there are no known analytical obstructions to such a control.

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How complex is the classification of transitive homeomorphisms?

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We prove that there is a Borel reduction of the homeomorphism equivalence relation (ER) of zero-dimensional compact metrizable spaces to the conjugacy ER of transitive Cantor set homeomorphisms. We also prove that the homeomorphism ER of absolute retracts (i.e. retracts of the Hilbert cube) is Borel reducible to the conjugacy ER of transitive Hilbert cube homeomorphisms.

By using already known results about the complexity of the above mentioned homeomorphism ERs we identify the exact complexity of conjugacy ERs of both transitive Cantor set maps and transitive Hilbert cube maps. Both the results are achieved by using essentially the same technique. Further, we discuss the problem of classification minimal maps.

Separable Lindenstrauss spaces whose duals do not contain weak* closed convex unbounded sets having the AFPP

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We characterize all separable L_1-preduals X for which every weak* closed convex unbounded set in X^* lacks the approximate fixed point property for nonexpansive mappings. Our result improves and completes a previous result in field, where this property was studied for $C(\alpha)$ and $C_0(\alpha)$ spaces with α an infinite countable ordinal as well as for ℓ_1-predual hyperplanes in c.

Lattice RDEs, mirroring, and an interesting functional equation

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This is the joint work with J. Hesoun and P. Stehlík (University of West Bohemia, Pilsen). We provide a complete characterization of equivalence classes of unbounded asymmetric stationary solutions of lattice reaction-diffusion equations which diverge in either one, or both spatial tails to infinity. Our main tool is an iterative (functional) mirroring technique which could be applicable to other problems related to lattice equations. Specifically, we reveal a natural relationship of lattice equations with an interesting functional equation which involves an unknown function and its inverse.

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On solution manifolds for differential equations with state-dependent delay

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Differential equations with state-dependent delays define a semiflow of continuously differentiable solution operators in general only on the associated *solution manifold* in the Banach space $C^1([-h,0],\mathbb{R}^n)$. For a prototypic example we develop a new proof that its solution manifold is diffeomorphic to an open subset of the subspace given by $\phi'(0)=0$, without recourse to a restrictive hypothesis about the form of delays which is instrumental in earlier work on the nature of solution manifolds. The new proof uses the framework of algebraic-delay systems.

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Nodal solutions for a class of coupled elliptic equations

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We report results on the existence of nodal solutions for a class of coupled nonlinear elliptic equations, in particular on multiplicity results of radial nodal solutions that share prescribed nodal data in the repulsive regime. The methods are minimax constructions in the presence of invariant sets of the associated gradient or heat flows. The results show a rich structure of nodal solutions for these equations.

Asymptotic stability of Markov operators in metric spaces

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The aim of this talk is to present recent results concerning the existence of an invariant measure for Feller semigroups acting on the space of bounded Borel functions on a metric space with applications to studying asymptotic stability of such semigroups. Our results complement the classical ones obtained by A. Lasota and J.A. Yorke [3] in proper spaces, and by T. Szarek [4] in Polish spaces.

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Infinitely many nonradial positive solutions for multi-species nonlinear Schrodinger systems in $\mathbb{R}^N$

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In this talk, I will report our recent results, based on the joint work of Doctor Tuoxing Li and Professor Jucheng Wei, on the multi-species nonlinear Schrodinger systems in RN. By Ljapunov-Schmidt reduction arguments, we construct infinitely many nonradial positive solutions of the these system under some mild assumptions on potentials and coupling parameters, without any symmetric assumptions on the limit case of the above system. Our result, giving a positive answer to the conjecture of Pistoia and Vaira in [Pistoia-Vaira, Comm. PDEs, 2022] and extending the results in [Peng-Wang, ARMA, 2013] and [Pistoia-Vaira ,Comm. PDEs, 2022], reveals new phenomenon for the two-species in dimension two and is almost optimal for the coupling parameters.

Keywords: nonlinear Schrodinger systems, infinitely many positive solutions, reduction method, min-max argument.

AMS Subject Classification 2010: 35B09; 35B33; 35B40; 35J20.

Optimization problems and parabolic equations on lower dimensional structures

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I will discuss the problem of finding an optimal conductivity tensor field minimizing thermal compliance for a given balanced distribution of heat sources and sinks. The heat sources are expressed as objects more general than Radon measures. The solutions to the optimization problem are quite explicitly expressed in terms of the data. The work is based on a foundation made by Bouchitté and Buttazzo. I will also discuss the related problems concerning solving the heat equation on lower dimensional structures in $\mathbb{R}^3$ which are formed by a finite union of one and two dimensional components.

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Typically periodic optimization in ergodic optimization

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Ergodic optimization is the study of problems relating maximizing invariant measures and maximum ergodic averages. In ergodic optimization theory, one important problem is the typically periodic optimization (TPO) conjecture. This conjecture was proposed by Mañé[6], Hunt, Ott and Yuan[4, 7] in 1990s, which reveals the principle of least action in dynamical system settings. To be more precise, TPO indicates that when the dynamical system is suitably hyperbolic and the observable is suitably regular, then the maximizing measure is "genetically" supported on a periodic orbit with relatively low period. In the setting of uniformly open expanding maps with Lipschitz/Holder observables, TPO was obtained in topological genetic sense by Contreras [3]in 2016. In this series of talks, I will report several recent progresses on understanding TPO conjecture both in probabilistic and topological sense, and for more general hyperbolic systems. These are joint works with Jairo Bochi[1], Ding and Li [2], and Huang Wen, Zeng Lian, Xiao Ma, Leiye Xu [5].

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Coincidence points of (n, m)-valued pairs of maps of a circle

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Given sets X, Y and $n \in \mathbb{N}$, a map $f\colon X \multimap Y$ is n-valued, if for every $x \in X$ the image $f(x)$ has cardinality n [3]. A *graph intersection point* of a pair of multivalued maps f and g is defined as a point $(x, y) \in X \times Y$ for which $f(x) \cap g(x) \neq \emptyset$, while a *domain coincidence point* is the x-coordinate of some point of the above intersections [1, 2]. In this talk we consider (n, m)-valued pairs of maps $f, g\colon S^1 \multimap S^1$ and study the relations between the number of domain coincidence points and the number of their graph intersection points.

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Normal forms, invariant manifolds and Lyapunov theorems

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We present an approach to Lyapunov theorems about center for germs of analytic vector fields based on the Poincaré–Dulac and Birkhoff normal forms. Besides new proofs of three Lyapunov theorems we prove their generalization: if the Poincaré–Dulac normal form indicates the existence of a family of periodic solutions then such family really exists. We also present new proofs of Weinstein and Moser theorems about lower bounds for the number of families of periodic solutions; here, besides the normal forms, some topological tools are used, the Poincaré–Hopf formula and the Lusternik–Schnirelmann category on weighted projective spaces.